The Optimal Tradeoff between the Probability and Magnitude of Fines

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Fines are used in a variety of situations to control activities which impose external costs.[1] Examples of such activities include polluting the air, speeding or double parking, evading taxes, littering a highway, and attempting to monopolize an industry.[2] If it were costless to "catch" or "observe" individuals (or firms) when they engage in an externality creating activity, presumably everyone would be caught and fined an amount equal to the external cost of the activity. This is simply the traditional Pigouvian tax solution. Individuals would then engage in the activity only if their private benefits exceed the external cost.

However, in most situations it is difficult or costly to catch individuals who impose external costs. If, as a result, individuals are caught with probability less than one, it is often observed that the fine could be raised to a level such that, as in the Pigouvian solution, individuals would engage in the activity only if their private gains exceed the external cost. Since this can apparently be done for any given probability of catching individuals and since it is normally costlier to catch a larger fraction of those engaging in the activity, it is frequently argued that the probability should be as low as possible. The only constraint on lowering the probability that is recognized is the inability of individuals to pay the fine; thus, the optimal fine implied by this argument equals an individual's wealth. Note that this reasoning applies regardless of the external cost of the activity.[3]

This view of the optimal tradeoff between the probability and magnitude of fines does not seem realistic. Individuals are rarely if ever fined an amount approximating their wealth, especially for activities which impose relatively small external costs.

The present paper points out an error in this view and suggests an explanation of the choice of the probability and fine which seems consistent with reality. The mistake is that the view does not properly take into account the possibility that individuals may be risk averse. The possibility of risk aversion does *not* imply that individuals cannot be induced to make the same decision about engaging in the activity as they would under the Pigouvian solution. (For any given probability, the fine can be lowered from the level at which its expected value equals the external cost to a level, reflecting risk aversion, such that only those individuals for whom the private gains exceed the external costs engage in the activity.) Rather, the error is that the view does not take into account the risk that is borne by those who do engage in the activity. This risk is present whenever those individuals have to pay a fine with a (positive) probability less than one.

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[1]Civil fines are used routinely by federal, state, and local governments. For example, the widespread and increasing use of money penalties by federal administrative agencies is documented by Harvey Goldschmid. Criminal fines are, of course, also used extensively (see Alec Samuels).

[2]Similar situations arise in a private context, for example, in monitoring the effort of employees. Although fines are not used often, other monetary incentives exist, such as promotion opportunities.

[3]This argument was apparently first made explicit by Gary Becker (pp. 183–85, 191–93) and has been accepted by many others. See, for example, Kenneth Elzinga and William Breit (ch. 7), William Landes and Richard Posner (pp. 10–11), and Richard Posner (pp. 164–72). Related issues have also been considered by Serge-Christophe Kolm, Gordon Rausser and Richard Howitt, Balbir Singh, T. N. Srinivasan, George Stigler, Gordon Tullock (pp. 151–68), and Laurence Weiss.

The paper is organized as follows. Section I describes the problem of maximizing social welfare when individuals choose whether to participate in an externality creating activity. Section II shows that, as would be expected, if individuals are risk neutral, the optimal probability is as low as possible and the fine is as high as possible. However, the assumption of risk neutrality seems implausible in situations in which individuals would face the risk of losing all of their wealth. Section III proves two propositions if individuals are risk averse. First, if the cost of catching individuals is sufficiently small, the optimal probability equals one. This is true because the disutility from bearing the risk of being caught and fined outweighs the potential savings from a reduction in the probability. Also, when the optimal probability is one, the optimal fine equals the private gain of those who engage in the activity. Second, if it is optimal to control the activity at all, then, regardless of how costly it is to catch individuals, it may never be optimal to catch them with a very low probability and to fine them much more than the external cost. This is true because doing so would lower utility due to risk bearing and could more than offset the benefits from controlling participation in the activity. Section IV shows that it could not be beneficial to allow individuals to reduce the risk of being fined by the purchase of insurance against fines. This is true because the government could achieve the same result by lowering the fine. Section V illustrates the main points of the paper with some hypothetical calculations of the optimal probability and fine for double parking. Section VI discusses some additional features of the problem of determining the optimal tradeoff between the probability and magnitude of fines.

Although this paper does not take considerations of justice into account, it is obvious that they are relevant to determining the desired probability and fine. For example, fining individuals far in excess of the external cost they impose on society may be thought of as unfair, or catching only a small fraction of those who impose the external costs may be seen as arbitrary. These considerations appear to complement the conclusions of this paper if individuals are risk averse.

I. The Model

Individuals, who are assumed to be identical, are faced with the choice of whether to engage in an externality creating activity. What an individual would gain from engaging in it depends on random factors but is known to him before he has to make the decision.[4] If an individual chooses to engage in the activity, there is a chance that he will be caught and fined. The government finances its efforts to catch individuals by a per capita tax. Fines collected from those who are caught are used to reduce this tax. Individuals are allowed to insure fully against the risk of bearing the external cost, but are not allowed to insure at all against the risk of being fined.

To be more specific, define the following notation.

U = function giving utility of wealth. This function is assumed to reflect risk neutrality or risk aversion.

y = initial wealth.

a, b = possible gains from engaging in the activity ($a < b$). Although the gains are treated as nonmonetary in nature, they are assumed to have monetary equivalents, a and b.

q = probability that the gain would be a ($0 < q < 1$).

n = proportion of the population who engage in the activity. The decision to engage in the activity is made by individuals and is described below.

e = external cost created whenever an individual engages in the activity. Each individual in the population is assumed to be equally likely to bear this external cost.

π = per capita insurance premium for full coverage against the external cost. The premium equals the expected external cost per capita,[5]

$$\pi = ne \tag{1}$$

f = fine collected from an individual who engages in the activity and is caught. It is

[4]For example, the gain from double parking is easily imagined to vary with circumstances.

[5]Since risk-neutral individuals would be indifferent between having the insurance and bearing the risk, it does not affect the analysis to assume that they pay the premium.

assumed that the fine does not depend on the individual's private gain because it is too costly (or impossible) for the government to determine the gain.

p = probability that an individual who engages in the activity is caught. The probability of being caught is also assumed to be independent of the individual's gain.

$c(p, \lambda)$ = per capita cost of maintaining p as the probability of catching those who engage in the activity, where $c_p > 0$;[6] λ is a shift parameter of the cost function, where $c_\lambda > 0$ and $c(p, 0) = 0$.[7]

t = per capita taxes. The government sets the tax to finance the cost of catching individuals in excess of the fine revenue collected,[8]

$$(2) \qquad t = c(p, \lambda) - npf$$

An individual would engage in the activity if the expected utility of doing so—taking into account the gain and the probability of having to pay the fine—exceeds the utility of his initial wealth. Thus, an individual who would gain a would engage in the activity if

$$(3) \quad (1-p)U(y-t-\pi+a) + pU(y-t-\pi+a-f) > U(y-t-\pi)$$

Similarly, an individual who would gain b would engage in it if[9]

$$(4) \quad (1-p)U(y-t-\pi+b) + pU(y-t-\pi+b-f) > U(y-t-\pi)$$

The proportion of the population engaging in the activity, n, is determined by (3) and (4).

Let EU_a and EU_b be, respectively, the expected utility of an individual, conditional on whether he gains a or b from engaging in the activity and where the decision whether to engage in it is made according to (3) or (4). Then the (unconditional) expected utility of an individual, calculated before he knows whether his potential gain is a or b, is

$$(5) \qquad EU = qEU_a + (1-q)EU_b$$

The problem discussed here is the determination of the probability and fine that maximize expected utility.[10] Formally, the problem is to choose p and f to maximize (5), where (3) and (4) determine EU_a and EU_b in terms of p, f, t, and π, and where (1) and (2) determine π and t. In order to make this problem interesting, it is assumed that if it were costless to control individual behavior, then expected utility would be maximized if those who would gain the larger amount engage in the activity, but not those who would gain the smaller amount. In other words, $a < e < b$.[11]

Before proceeding it is necessary to discuss the *threshold probability*, the highest probability below which it is impossible to deter individuals from engaging in the activity. Such a probability exists because there is a limit to how much an individual can be fined.[12] More precisely, let w be an individual's wealth, g the potential gain, and $\bar{p}(w, g)$ the threshold probability. Since g is nonmonetary in nature, if an individual engages in the activity and is fined the maximum amount w, his utility is $U(g)$. The threshold probability is determined by

$$(6) \quad (1-\bar{p})U(w+g) + \bar{p}U(g) = U(w)$$

since with the highest possible fine, $\bar{p}$ is such that the individual would be indifferent between engaging and not engaging in the activity. Obviously, if p is less than $\bar{p}$, the individual would engage in the activity since the fine cannot be raised.

[6]Subscripts are used to denote partial derivatives.

[7]The conclusions of this paper would not be affected if the cost also depended on the number of persons who engage in the activity.

[8]Per capita fine revenue is treated as deterministic; the justification is, of course, the law of large numbers.

[9]If individuals are indifferent between engaging and not engaging in the activity, it will be clear from context whether or not it is assumed that they engage in it.

[10]This problem is equivalent to that of maximizing the sum of utilities for a population with a proportion q of individuals who would gain a from engaging in the activity and a proportion $(1-q)$ who would gain b.

[11]If $e < a$, then there is no reason to discourage anyone from engaging in the activity. If $e > b$, then all should be discouraged if it is costless to do so; when it is costly to catch individuals, then the analysis can be easily developed from what follows.

[12]The existence of this probability was first noted by Michael Block and Robert Lind.

Solving for the threshold probability from (6) gives[13]

$$(7)\qquad \bar{p}(w,g) = \frac{U(w+g) - U(w)}{U(w+g) - U(g)}$$

Note that the higher the private gain, the higher the threshold probability,

$$\begin{aligned}(8)\quad \bar{p}_g = {} & [U'(w+g)[U(w) - U(g)] \\ & + U'(g)[U(w+g) - U(w)]] \\ & \div [U(w+g) - U(g)]^2 > 0\end{aligned}$$

This makes sense because it should be harder to prevent individuals from engaging in the activity if the gain from doing so rises.

II. Individuals are Risk Neutral

In this case, the optimal tradeoff between the probability and magnitude of fines is described by the following proposition.

PROPOSITION 1: *Suppose that individuals are risk neutral and that it is optimal to control the externality creating activity. Then the optimal probability is as low as possible (equal to the threshold probability of those who gain the least) and the optimal fine is as high as possible (equal to an individual's wealth).*[14]

(a) *This result is true no matter how low the cost of catching individuals.*

(b) *At the optimum, only those individuals whose private gains exceed the external cost engage in the activity.*

PROOF:

Let p^* and f^* be the optimal values of the probability and fine and suppose that $p^* > 0$ (i.e., it is optimal to control that activity). The proposition then states the $p^* = \bar{p}(y - t - \pi, a)$ and $f^* = y - t - \pi$.

Since $p^* > 0$, the *a*s (those who would gain a) do not engage in the activity and the *b*s (those who would gain b) do engage in it: otherwise there are three possibilities to consider. If all individuals engage in the activity, it would clearly be better to spend nothing to control the activity (i.e., set $p = 0$), contradicting the optimality of p^*.[15] If only the *a*s engage in the activity, it would again be better to spend nothing to control the activity since then the *b*s, for whom the private benefit exceeds the external cost, would also be induced to engage in the activity.[16] If neither the *a*s nor the *b*s engage in the activity, it is possible to alter only the fine so as to induce the *b*s but not the *a*s to engage in the activity.[17]

Suppose that $p^* < \bar{p}(y - t - \pi, a)$. Then by definition the *a*s would engage in the activity, contradicting the result of the previous paragraph.

Suppose that $p^* > \bar{p}(y - t - \pi, a)$. Since individuals are risk neutral, let $U(w) = w$ without loss of generality. Then, since the *a*s are not engaging in the activity and the *b*s are, it follows that

$$\begin{aligned}(9)\quad & (1 - p^*)(y - t - \pi + a) \\ & + p^*(y - t - \pi + a - f^*) \\ & \leqq y - t - \pi\end{aligned}$$

$$\begin{aligned}(10)\quad & (1 - p^*)(y - t - \pi + b) \\ & + p^*(y - t - \pi + b - f^*) \\ & \geqq y - t - \pi\end{aligned}$$

which, respectively, reduce to

[13] If $g \geq w$, define $\bar{p}(w,g) = 1$. In this case an individual would *always* choose to engage in the activity, since the gain is at least as high as the maximum potential loss through a fine. If $0 < g < w$, then $0 < \bar{p}(g, w) < 1$. In this case, there exists a probability and a fine which could deter individuals from engaging in the activity. It will be assumed that the latter case applies.

[14] It is clearly optimal to control the activity if the cost of catching individuals (at the threshold probability of those who gain the least) is sufficiently low.

[15] Since at p^* the *a*s and *b*s engage in the activity, $\pi = e$ and $t = c(p^*, \lambda) - p^*f$. Therefore, from (5), $EU = y - e + qa + (1-q)b - c(p^*, \lambda)$ but at $p = 0$, $EU = y - e + qa + (1-q)b$ which is larger since $c(p^*, \lambda) > 0$.

[16] Since only the *a*s engage in the activity, $\pi = qe$ and $t = c(p^*, \lambda) - p^*f^*q$. Therefore, from (5), $EU = y - qe - c(p^*, \lambda) + qa$. But at $p = 0$, $EU = y - e + qa + (1-q)b$. The latter exceeds the former since $-e + (1-q)b > -e + (1-q)e = -qe > -qe - c(p^*, \lambda)$.

[17] If neither engage in the activity, $\pi = 0$ and $t = c(p^*, \lambda)$. Therefore from (5), $EU = y - c(p^*, \lambda)$. It is clear that there is an $f^0 < f^*$ such that only the *b*s engage in the activity. Then $\pi = (1-q)e$ and $t = c(p^*, \lambda) - (1-q)p^*f^0$. From (5), $EU = y - c(p^*, \lambda) + (1-q)(b-e)$, which is higher than expected utility when neither engage since $b > e$.

$$(11) \qquad a \leqq p^* f^*$$

$$(12) \qquad b \geqq p^* f^*$$

There are two cases to consider. If $a < p^* f^*$, then holding f^* fixed, lower p^* slightly to p' so as to satisfy $a < p' f^*$. Then, since $b > p' f^*$, it is still true that (11) and (12) are satisfied at p' and f^*, that the as do not engage in the activity and that the bs do. Therefore at p' and f^*, π has not changed but t has changed to t', where

$$(13) \qquad t' = c(p', \lambda) - (1-q) p' f^*$$

Next subtract expected utility (5) at p^* and f^* from that at p' and f^*, to get

$$\begin{aligned}(14) \quad & [q(y - t' - \pi) + (1-q) \\ & (y - t' - \pi + b - p' f^*)] \\ & - [q(y - t - \pi) + (1-q) \\ & (y - t - \pi + b - p^* f^*)] \\ & = (t - t') + (1-q)(p^* - p') f^* \\ & = c(p, \lambda) - c(p', \lambda) > 0\end{aligned}$$

Thus, p^* and f^* could not have been optimal. The other case is $a = p^* f^*$. In this case,

$$\begin{aligned}(15) \qquad f^* &= a/p^* < a/\bar{p}(y - t - \pi, a) \\ &= y - t - \pi\end{aligned}$$

Thus, it is possible to raise f^* slightly to $f' < y - t - \pi$ and to lower p^* to $p' = p^* f^* / f'$. Since $p' f' = p^* f^*$, (11) and (12) continue to hold at p' and f', π does not change and taxes fall by

$$\begin{aligned}(16) \quad & [c(p^*, \lambda) - (1-q) p^* f^*] \\ & - [c(p', \lambda) - (1-q) p' f'] \\ & = c(p^*, \lambda) - c(p', \lambda) > 0\end{aligned}$$

Since taxes fall and nothing else has changed, expected utility must have risen, so that again p^* and f^* could not have been optimal. Hence $p^* = \bar{p}(y - t - \pi, a)$ as claimed and, by definition of the threshold probability, $f^* = y - t - \pi$; otherwise the as would engage in the activity.

III. Individuals are Risk Averse

In this case, the optimal tradeoff between the probability and magnitude of fines is described by two propositions. The first says that if the cost of catching individuals, λ, is sufficiently small, the optimal probability equals one. (Recall that if $\lambda = 0$, it is costless to catch individuals who engage in the activity and that as λ increases it becomes more costly to catch individuals for any p.) As noted in the introduction, this is because the disutility from bearing the risk of being caught is more important than the savings in the cost of catching individuals. When the optimal probability equals one, the optimal fine equals the private gain b of those who engage in the externality generating activity. (Although any fine between the external cost e and b would lead those individuals to engage in the activity, a fine of b is used in order to transfer income from those who have favorable opportunities for gains to those who do not.) The second proposition says that as λ increases, the optimal probability may never become as low as the threshold probability before it drops to zero and the optimal fine may never become as high as an individual's wealth. As noted in the introduction, this is because the use of a smaller probability and a higher fine may lower utility due to risk bearing and more than offset the benefits from controlling participation in the activity. These propositions are illustrated in Figure 1, which relates the optimal probability p^* to λ.[18] The diagram also shows the threshold probability, $\bar{p}(y - t - \pi, a)$, of individuals who would gain a from engaging in the activity.

PROPOSITION 2: *Suppose that individuals are risk averse. Then if the cost of catching them is sufficiently low, the optimal probability equals one and the optimal fine equals the private gain of those who engage in the externality creating activity.* (See the Appendix for the proof of Proposition 2.)

The proof consists of several steps. It is first shown that when $\lambda = 0$, $p^* = 1$. Then, in three

[18]The probability p^* does not necessarily decline with λ everywhere, although one would expect this to be the typical case. As λ rises, *ceteris paribus*, the tax necessary to maintain p at any level would rise, lowering wealth and raising absolute risk aversion (assuming it is decreasing with wealth). Since this would increase the disutility of bearing the risk of being caught, there would be a tendency for p^* to rise.

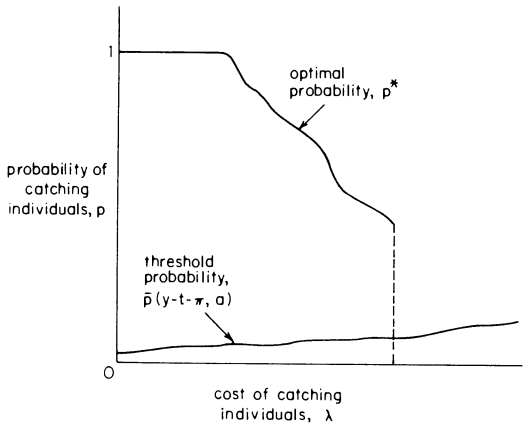

FIGURE 1

steps, a continuity argument is used to show that as $\lambda \rightarrow 0$, $p^* \rightarrow 1$. By a similar argument, it is shown that as $\lambda \rightarrow 0, f^* \rightarrow b$, where f^* is the optimal fine. Finally, these facts are used to prove the proposition.

PROPOSITION 3: *Suppose that individuals are risk averse. Then, as the potential gains from controlling the externality creating activity approach zero (i.e., as $b \rightarrow a$), the (minimum positive) optimal probability approaches one and the optimal fine approaches the private gain of those who engage in the activity.*

The reasoning in the proof is similar to that in Propositions 1 and 2 and will only be sketched below. Referring to Figure 2, the

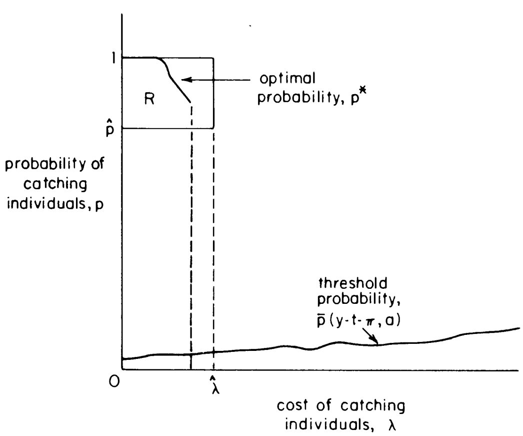

FIGURE 2

proof shows that given any small rectangle R (with northwest corner at (0, 1)), the positive portion of the $p^*(\lambda)$ schedule cannot be outside R if b is sufficiently close to a.

PROOF:

Let the coordinates of the southeast corner of R be $(\hat{\lambda}, \hat{p})$. If $p^*(\lambda) > 0$ is ever outside R, either $\lambda > \hat{\lambda}$ or $p^*(\lambda) < \hat{p}$. Suppose first that $\lambda > \lambda$. Since $p^*(\lambda)$ is optimal, it must be at least equal to the threshold probability at λ, namely $\bar{p}(y - t - \pi, a)$, where t and π are implicitly determined by λ. Since this threshold probability is bounded away from zero,[19] expenditure on catching individuals is positive. Therefore EU is bounded away from $V(1, 0)$. (See proof of Proposition 2 for definition of V.) But clearly as $b \rightarrow a$, EU approaches $V(1,0)$ if the activity is not controlled. Therefore, $p^*(\lambda)$ could not be optimal if b is sufficiently close to a.

Now assume that $p^*(\lambda) < \hat{p}$. Since $p^*(\lambda) > 0$ and is optimal, only the bs engage in the activity. But the fine is bounded from below, for otherwise the as would engage in the activity. Thus (refer to step (ii) of the proof to Proposition 2), disutility due to risk bearing is imposed on the bs; EU is therefore bounded away from $V(1, 0)$, which leads to a contradiction by the argument of the previous paragraph.

To show that the fine approaches b, note first that if the bs engage in the activity, the expected fine must be less than or equal to b, that is, the fine must be less than or equal to $b + b(1 - p)/p$. Since, as $b \rightarrow a$, the minimum positive optimal probability approaches 1, the fine is bounded above by $\lim_{p \rightarrow 1} b + b(1 - p)/p = b$. On the other hand, since the as do not engage in the activity, the fine must approach a limit at least equal to a, since the minimum positive probability approaches 1. Thus, as $b \rightarrow a$, the optimal fine approaches b (and a).

IV. Insurance Against Fines

Insurance against fines is in general not allowed in this country. However, because

[19]It is clear that the threshold probability for the as is bounded away from zero since income is bounded from above.

such insurance would reduce the disutility of bearing the risk of being fined for individuals who engage in the externality creating activity, it might seem that the insurance could be socially desirable under some conditions.[20] This section shows that such insurance cannot be beneficial because, as noted in the introduction, the government could achieve the same result by lowering the fine.

The type of insurance against fines to be considered is coverage which individuals purchase before they know their private gains from engaging in the activity. In other words, it is assumed that it would be impractical for individuals to buy insurance once they know their private gains.

Introduce the following notation:

h = insurance coverage against fines.

z = premium for coverage against fines.

It is assumed that insurance is sold either by competitive firms or directly by a public agency and, in either case, that the premium is actuarially fair. Thus, since n is the fraction of individuals who engage in the activity,

$$z = nph \tag{17}$$

To see that insurance cannot increase expected utility, consider an equilibrium in which individuals purchase coverage h against fines. If g is the gain from engaging in the activity, an individual would engage in it if

$$\begin{aligned}(1-p)U(y-t-z-\pi+g) \\ + pU(y-t-z-\pi+g-f+h) \\ > U(y-t-z-\pi)\end{aligned} \tag{18}$$

Now consider the outcome if individuals are not allowed to purchase insurance against fines and the tax is changed from t to $\bar{t} = t + z$, the fine from f to $\bar{f} = f - h$, and nothing else is changed. Then an individual would engage in the activity if

$$\begin{aligned}(1-p)U(y-\bar{t}-\pi+g) \\ + pU(y-\bar{t}-\pi+g-\bar{f}) \\ > U(y-\bar{t}-\pi)\end{aligned} \tag{19}$$

[20]Closely related questions which arise in connection with liability insurance are discussed by Shavell.

which is clearly equivalent to (18). Thus individual behavior and expected utility in this case are identical to that when individuals purchased coverage h. To verify that the budget balances, note that it must have been true in the equilibrium when h was purchased that $t = c - npf$. By this and (17),

$$\begin{aligned}\bar{t} &= t + z = c - npf + nph \\ &= c - np(f-h) = c - np\bar{f}\end{aligned} \tag{20}$$

which completes the argument.

V. Double Parking—A Numerical Example

The optimal tradeoff between the probability and magnitude of fines if individuals are risk averse may be illustrated by a numerical example. The example concerns the control of double parking in a hypothetical city in which the private gains to residents from double parking usually are exceeded by the congestion costs imposed, but occasionally are not. The example is described by the following data:

100,000 = population of the city

10,000 = number of locations where a resident could double park

25 = number of locations which each policeman can check per hour (and, if necessary, at which he must write a ticket)

s = wage of a policeman per hour

$\log w$ = utility of wealth w for each resident

\$10,000 = initial wealth of each resident

\$5.00 = congestion costs from double parking

\$4.50, \$50.00 = possible private gains to each resident from double parking

.90 = probability that the private gain is \$4.50.

In order to catch individuals who double park with probability p, $10{,}000p$ locations must be checked, $10{,}000p/25 = 400p$ policemen must be employed, and a total of $400ps$ must be spent on enforcement. If the fine is f,

TABLE 1—THE CONTROL OF DOUBLE PARKING

Wage (Per Hour)	Optimal Probability	Optimal Fine	Total Enforcement Expenditures at Optimal Probability	Threshold Probability	Total Enforcement Expenditures at Threshold Probability
\$3.00	.09	\$50.10	\$108	.00006	\$0.07
\$5.00	.07	\$64.10	\$140	.00006	\$0.12
\$7.50	.06	\$74.80	\$180	.00006	\$0.18
\$10.00	.05	\$89.80	\$200	.00006	\$0.23
\$15.00	.04	\$111.90	\$240	.00006	\$0.35
\$25.00	.03	\$149.00	\$300	.00006	\$0.58

then total fine revenue is pf times the number of residents who double park. Per capita taxes equal total expenditures on enforcement minus total fine revenue, all divided by 100,000. Per capita congestion costs are \$5.00 times the number of people who double park, divided by 100,000. Given his wealth net of taxes and congestion costs, his private gain, and the probability and magnitude of the fine, each resident decides whether to double park (see Section I). The expected utility of a typical resident may then be calculated.

The probability and fine which maximize expected utility of the residents were computed for various values of the policemen's wage. The results may be summarized as follows:

(a) If the wage is less than \$2.83, the optimal probability is 1 and the optimal fine is \$50. This result, which illustrates Proposition 2, shows that when policemen can be cheaply hired, it is best to employ a sufficient number to catch all individuals who double park. Total expenditures on enforcement may be as high as \$1,132 (when the wage is \$2.83). In contrast, the government could use the maximum possible fine of \$10,000 with the threshold probability of .00006. This is the lowest probability which would deter those who would gain \$4.50 from double parking; however, those who would gain \$50 would double park. Although this system of enforcement would involve total expenditures of at most \$0.07, it is not optimal because of the high risk imposed on those who double park.

(b) As the wage rises above \$2.83, the optimal probability rapidly declines and the fine increases. Table 1 reports the results for several reasonable values of a policeman's wage. As the wage increases from \$3 to \$25, the optimal probability falls from .09 to .03, the optimal fine rises from \$50.10 to \$149.00, and total enforcement expenditures increase from \$108 to \$300. In contrast, if the fine were incorrectly set at its maximum \$10,000, and the probability were set at the threshold .00006, then total expenditures on enforcement would never exceed \$0.58.

(c) As the wage increases to extremely high levels, the optimal probability declines to a value slightly above the threshold probability and the optimal fine rises to approximately \$9,000. Only when the wage exceeds \$200,000 *per hour* is the optimal policy to do nothing about double parking. This illustrates Proposition 3 since, when it is optimal to control double parking, the optimal fine never reaches \$10,000 and the optimal probability never becomes as low as the threshold probability. If the parameters of the example were different, the wage at which it first becomes optimal to do nothing about double parking might be much lower; as a result, the maximum optimal fine might be much lower and the minimum positive optimal probability much higher. For instance, this would be the case if the lower private gain from double parking were just below the congestion costs imposed.

VI. Concluding Remarks

The model used here abstracted from a variety of considerations relevant to the determination of the optimal probability and fine. Several of these are now mentioned.

It was assumed that the private gains from engaging in the externality creating activity could not be observed. However, in some contexts the private gains might be identifiable at little cost. If this is the case, the fine (and possibly the probability) could depend on the private gain. Those who would gain less than the external cost could be discouraged from engaging in the activity by setting their fine sufficiently high, and those who would gain more than the external cost could be induced to engage in the activity by setting their fine sufficiently low, possibly at zero. Therefore, the optimal probability can be lowered since disutility due to risk bearing can be reduced.

It was also assumed in the model that individuals who did not engage in the activity were never mistakenly fined. If this possibility had been taken into account, the conclusions of this paper would be reinforced, since all individuals would then bear the risk of paying a fine.

Furthermore, it was assumed that if individuals engaged in the activity at all, they did so at a particular level. A more general model would allow individuals to engage in the activity at varying levels. Similarly, it was assumed that the distribution of private gains was discrete, whereas in general the distribution might be continuous. Neither of these extensions would affect the basic results of this paper. All that is required for the results is that, given the optimal probability and fine, some risk-averse individuals generate externalities at a positive level and are subject to the risk of having to pay a fine.

Finally, it was assumed that individuals had the same level of wealth. However, differences in wealth may be important in many situations. Suppose that absolute risk aversion decreases with wealth and that the probability and fine cannot be made to depend on wealth. Then, *ceteris paribus,* any given probability and fine would be less likely to discourage a wealthy individual from engaging in the activity than a poor one. As a result, in the optimal solution, some wealthy individuals might be underdeterred—induced to engage in the activity even though their private gains are less than the external cost—and some lower income individuals who are able to pay the fine might be overdeterred. However, some poor individuals who are unable to pay the fine might be underdeterred.

Appendix

PROOF of Proposition 2:

Let p^* and f^* be the optimal values of the policy parameters. Define $V(p, \lambda)$ as the maximum expected utility (5) given p and λ.

(i) $V(1, 0) > V(p, 0)$ for $p < 1$: If, given p and the associated optimal fine, f, all individuals engage in the activity, then $\pi = e$ and $t = -pf$, so that

$$\text{(A1)} \quad \begin{aligned} EU_a &= (1 - p)U(y + pf - e + a) \\ &\quad + pU(y + pf - e + a - f) \\ &\leqq U(y + pf - e + a - pf) \\ &= U(y - e + a) \end{aligned}$$

The inequality follows since U is concave (individuals are risk averse). Similarly,

$$\text{(A2)} \quad EU_b \leqq U(y - e + b)$$

Hence

$$\text{(A3)} \quad \begin{aligned} V(p, 0) &\leqq qU(y - e + a) \\ &\quad + (1 - q)U(y - e + b) \\ &\leqq U(y - e + qa \\ &\qquad + (1 - q)b) \\ &= U(y + (1 - q)(b - e) \\ &\qquad + q(a - e)) \end{aligned}$$

On the other hand, if the probability equals 1 and the fine equals b, then the as will not engage in the activity and the bs will. Hence $\pi = (1 - q)e$, $t = -(1 - q)b$, and

$$\text{(A4)} \quad \begin{aligned} EU_a &= EU_b \\ &= U(y + (1 - q)(b - e)) \end{aligned}$$

so that, since $a < e$,

$$\text{(A5)} \quad \begin{aligned} V(1, 0) &\geqq U(y + (1 - q)(b - e)) \\ &> U(y + (1 - q)(b - e) \\ &\qquad + q(a - e)) \geqq V(p, 0) \end{aligned}$$

If at p and f *no* individuals engage in the activity, then $\pi = t = 0$, so

$$\text{(A6)} \quad V(p,0) = U(y) < U(y + (1-q)(b-e)) \leqq V(1,0)$$

If at p and f, only the as engage in the activity, then $\pi = qe$, $t = -qpf$; the following chain of inequalities holds:

$$\begin{aligned}\text{(A7)} \quad & V(p,0) \\ &= q[(1-p)U(y + q(pf - e) + a) \\ &+ pU(y + q(pf-e) + a - f)] \\ &+ (1-q)U(y + q(pf-e)) \\ &\leqq qU(y + q(pf-e) + a - pf) \\ &+ (1-q)U(y + q(pf-e)) \\ &\leqq U(y + q(pf-e) \\ &+ q(a - pf)) = U(y + q(a-e)) \\ &< U(y + (1-q)(b-e)) \leqq V(1,0)\end{aligned}$$

The first two inequalities follow from the concavity of U; the third inequality follows from $a < e < b$; and the last inequality follows from (A5).

The remaining possibility is that at p and f the as do not engage in the activity and the bs do engage in the activity. Then $\pi = (1-q)e$ and $t = -(1-q)pf$. In this case, raise the probability to 1 and lower the fine to a value f^0 such that the utility of the bs is unchanged if π and t are held constant; i.e.,

$$\begin{aligned}\text{(A8)} \quad & (1-p)U(y - t - \pi + b) \\ &+ pU(y - t - \pi + b - f) \\ &= U(y - t - \pi + b - f^0)\end{aligned}$$

At a probability equal to 1 and a fine of f^0, the fine revenue raised by the government is higher since

$$\text{(A9)} \qquad f^0 > pf$$

To see this, observe that the left-hand side of (A8) is less than $U(y - t - \pi + b - pf)$ since U is concave (and $f \geq a > 0$, for otherwise the as would engage in the activity), so that (A8) cannot hold unless the fine f^0 exceeds pf. It is also true that

$$\text{(A10)} \qquad f^0 \leqq b$$

since otherwise the left-hand side of (A8) is less than $U(y - t - \pi)$, contradicting the assumption that the bs choose to engage in the activity.

Now there are two cases to consider: $f^0 \geqq a$ and $f^0 < a$. In the former case, let per capita taxes be reduced by the amount

$$\text{(A11)} \qquad s_1 = (1-q)(f^0 - pf) > 0$$

which is positive by (A9). At a probability equal to 1, f^0, $\pi = (1-q)e$, and the new and lower taxes, it is still true that the as do not engage in the activity and the bs do, for $a \leqq f^0 \leqq b$. However, the as are better off than at $p < 1$ for after-tax income has risen by s_1; and the bs are better off because $U(y - t - \pi + b - f^0)$ is less than $U(y - t + s_1 - \pi + b - f^0)$. Consequently p and f could not have been optimal.

On the other hand, if $f^0 < a$, then at a probability equal to 1, the bs will still choose to engage in the activity; although the as would in fact also choose to engage in the activity, note that if they did *not*, their expected utility at a probability of 1 and f^0 (and $\pi = (1-q)e$, $t = -(1-q)pf$) would be identical to that at p and f. Now reduce taxes by s_1, as in the previous case. This raises both the as and bs expected utility assuming (temporarily) that the as do not engage in the activity. Expected utility, EU, is therefore

$$\begin{aligned}\text{(A12)} \quad & qU(y - t + s_1 - \pi) + (1-q) \\ &\cdot U(y - t + s_1 - \pi + b - f^0)\end{aligned}$$

Now at a probability of 1, raise the fine from f^0 to a. At this level, the as do not engage in the activity and the bs do, so that $\pi = (1-q)e$. Since per capita fine revenue rises from $(1-q)f^0$ to $(1-q)a$, per capita taxes can be reduced further by

$$\text{(A13)} \qquad s_2 = (1-q)(a - f^0) > 0$$

Define $s = s_1 + s_2$. Expected utility is therefore

$$\begin{aligned}\text{(A14)} \quad & qU(y - t + s - \pi) + (1-q) \\ &\cdot U(y - t + s - \pi + b - a)\end{aligned}$$

If (A14) exceeds (A12), it will have been demonstrated that EU can be made higher at a probability of 1 than at p, completing this step of the proof. But expected wealth in (A12) and (A14) is easily verified to be identical. Hence, since U is concave, and

$$\text{(A15)}\quad (y - t + s - \pi) > (y - t + s_1 - \pi)$$
$$(y - t + s - \pi + b - a) < (y - t + s_1 - \pi + b - f^0)$$

(A14) exceeds (A12).[21]

(ii) For any $\delta > 0$ there exists a $\gamma > 0$ such that if $p < 1 - \delta$, then $V(1, 0) - V(p, 0) > \gamma$: This step follows directly from the previous step. Recall that at $p < 1$ (and in particular at $p < 1 - \delta$) there were four possibilities. If all individuals engage in the activity, it was shown that $V(1, 0) > V(p, 0)$ by at least a positive amount, say γ_1, which is independent of p (see (A5)). If no individuals engage in the activity, or only the as engage in it, again $V(1, 0) > V(p, 0)$ by at least positive amounts γ_2 and γ_3, respectively, independent of p (see (A6) and (A7), respectively). Finally, if at p the as do not engage in the activity and the bs do, then it was shown that $V(1, 0) > V(p, 0)$. The first part of that demonstration involves raising the probability to one and lowering the fine to f^0; this generated surplus s_1 (see (A11)), which was distributed and raised expected utility. That argument implies that the size of s_1—and hence of the subsequent utility gain—is larger the smaller is p. Since p is no larger than $1 - \delta$, there exists a $\gamma_4 > 0$ serving as a lower bound for $V(1, 0) - V(p, 0)$ in this final case. Pick γ = minimum γ_i.

(iii) For any $\delta > 0$, there exists $\epsilon > 0$ such that if $\lambda < \epsilon$, the optimal probability, $p^*(\lambda)$, satisfies $p^*(\lambda) \geqq 1 - \delta$; in other words, $p^*(\lambda) \to 1$ as $\lambda \to 0$: Assume otherwise. Then for some $\delta > 0$, there exists a λ arbitrarily small such that $p^*(\lambda) < 1 - \delta$. It is easy to show that $V(1, \lambda) \to V(1, 0)$ as $\lambda \to 0$. Therefore, pick ϵ such that if $\lambda < \epsilon$, then $|V(1, \lambda) - V(1, 0)| < \gamma/2$. Now certainly $V(p, \lambda) \leqq V(p, 0)$ for all p and since for $p < 1 - \delta$, $V(1, 0) - V(p, 0) > \gamma$ (by step (ii)) it must be true that $V(1, 0) - V(p, \lambda) > \gamma$. Hence, if $\lambda < \epsilon$, $V(1, \lambda) - V(p, \lambda) > \gamma/2 > 0$. Therefore $V(p, \lambda)$ could not have been the maximum, a contradiction.

(iv) If $p^*(\lambda) = 1$, the optimal fine is b: If $p = 1$ and $f = b$, the bs engage in the activity, the as do not, and the utility of each individual is $U(y - c(1, \lambda) + (1 - q)(b - e))$. This fine must be optimal since it coincides with the first best solution to the problem.[22]

(v) $p^*(\lambda) = 1$ for all λ sufficiently low: It is first shown that $V_p(1, 0) > 0$. To do this, consider the function $W(p) = \max_f Y(p, f)$ where

$$\text{(A16)}\quad Y(p, f) = qU(y + (1 - q)(pf - e)) + (1 - q)[(1 - p)U(y + (1 - q)(pf - e) + b) + pU(y + (1 - q)(pf - e) + b - f)]$$

Thus $W(p)$ is the expected utility of individuals as a function of p when $\lambda = 0$ and f is selected optimally, but without constraining f to be such that the bs are induced to engage in the activity and the as discouraged from doing so. Therefore,

$$\text{(A17)}\qquad W(p) \geqq V(p, 0)$$

It may easily be verified that

$$\text{(A18)}\qquad W(1) = Y(1, b) = V(1, 0)$$

Thus, if $W'(1) > 0$, then $V_p(1, 0) > 0$. But since $W(1) = Y(1, b)$ and $Y_f(1, b) = 0$ (this is the first-order condition for optimal selection of f), it follows that

[21] In general, if $A = qU(w_1) + (1 - q)U(w_2)$ and $B = qU(w_3) + (1 - q)U(w_4)$, where $qw_1 + (1 - q)w_2 = qw_3 + (1 - q)w_4$, $w_1 < w_2$, $w_3 < w_4$, $w_3 > w_1$, and $w_4 < w_2$, then $A < B$ if U is concave. To prove this, define $C(t) = qU(w_1 + t) + (1 - q)U(w_2 - qt/(1 - q))$. Then $A = C(0)$ and $B = C(t_0)$ for some $t_0 > 0$. Now $C'(t) = q[U'(w_1 + t) - U'(w_2 - qt/(1 - q))] > 0$ as long as $w_1 + t < w_2 - qt/(1 - q)$ by concavity of U. Hence $B > A$ given our assumptions.

[22] Consider the first best (benevolent dictator's) problem of maximizing expected utility given that $p = 1$ subject only to the constraint that resources balance, $y - ne - c(1, \lambda) = qy_a + (1 - q)y_b$, where y_a is the wealth of the as and y_b that of the bs. It is clear that the solution involves ordering only the bs to engage in the activity and selecting y_a and y_b so that the marginal utilities of wealth for the as and bs are equal. Thus $n = (1 - q)$, $y_a = y - c(1, \lambda) + (1 - q)(b - e)$ and $y_b = y_a - b$ so that $EU_b = U(y_a - b + b) = U(y_a) = EU_a$.

$$\begin{aligned}(A19)\quad W'(1) &= Y_p(1,b) + Y_f(1,b)\frac{df}{dp} = Y_p(1,b)\\ &= q(1-q)bU'(y+(1-q)(b-e))\\ &\quad + (1-q)[U(y+(1-q)(b-e))\\ &\quad - U(y+(1-q)(b-e)+b)\\ &\quad + (1-q)bU'(y+(1-q)(b-e))] > 0\end{aligned}$$

The inequality follows because, by concavity of U, $U(y + (1-q)(b-e)) - U(y + (1-q)(b-e) + b) > -bU'(y + (1-q)(b-e))$.

Suppose it is not true that $p^*(\lambda) = 1$ for all λ sufficiently small. Then, since it was shown in step (iii) that $p^*(\lambda) \rightarrow 1$ as $\lambda \rightarrow 0$, there must exist a sequence $\{\lambda_i\}_{i=1}^{\infty}$ where $\lambda_i \rightarrow 0$, $p^*(\lambda_i) < 1$, and $p^*(\lambda_i) \rightarrow 1$. Since $p^*(\lambda_i) < 1$, it is an interior optimum so that $V_p(p^*(\lambda_i), \lambda_i) = 0$ for all i. Therefore,

$$(A20)\qquad \lim_{i\rightarrow\infty} V_p(p^*(\lambda_i), \lambda_i) = 0$$

On the other hand, by continuity of $V_p(p, \lambda)$,

$$(A21)\qquad \lim_{i\rightarrow\infty} V_p(p^*(\lambda_i), \lambda_i) = V_p(1,0) > 0$$

which is a contradiction.

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