A Model of Optimal Incapacitation

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One of the functions of the criminal sanctions of imprisonment and the death sentence is to prevent individuals from doing harm by removing them from the population.[1] This incapacitative function of sanctions is considered below in a model in which the amount of harm individuals cause each period that they are free is not influenced by the threat of sanctions (so as to abstract from the role of sanctions as a deterrent).[2]

The model is initially examined assuming that an individual's dangerousness (i.e., the harm he will do if free) remains the same each period of his life, and that the sanction is imprisonment. In this case, it is optimal to imprison an individual if his dangerousness exceeds a threshold equal to the per period social cost of imprisonment. Moreover, if it is optimal to imprison an individual at all, it will be best to do so for life. The optimal probability of apprehension is also determined.

The model is then extended in several ways. It is first supposed that the dangerousness of individuals declines with age. In this case, it is again optimal to imprison individuals if their dangerousness exceeds the per period cost of imprisonment, but it is optimal to release them if their dangerousness later falls below the threshold. It is next supposed that the dangerousness of individuals declines with time spent in prison due to a rehabilitative effect. In this case, it is optimal to imprison individuals beginning at a lower threshold of dangerousness than the per period cost of imprisonment (imprisonment is now socially more valuable) and to release them if their dangerousness becomes sufficiently low. Finally, it is supposed that a choice can be made between imprisonment and the death penalty, and the optimal choice is discussed.

I. Analysis

A. *Basic Model*

An equal number (normalized at one) of finite-lived individuals enter the population every period, so that in the steady state the population is comprised of equal numbers of individuals of each age cohort. An individual causes the same amount of harm every period that he is free, where the amount of harm varies by the individual. Individuals are apprehended each period with a probability, and if apprehended, may be imprisoned. Specifically, let h = harm done each period by an individual if not imprisoned; $0 \leqq h \leqq \bar{h}$; $f(h)$ = probability density of individuals of type h entering the population each period; n = number of periods that each individual lives; p = probability of apprehension of individuals each period; $s(h)$ = length of prison sentence imposed on an individual of type h if apprehended.

Society bears certain costs in apprehending and imprisoning individuals: let c = cost of imprisoning an individual per period,[3] $c > 0$; $e(p)$ = enforcement expenses associated with maintaining the probability of apprehension; $e'(p) > 0$, $e''(p) > 0$.

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[1] Note that monetary sanctions have no such function.

[2] While the deterrent effect of sanctions has of course been much investigated by economists (see fn. 5), the incapacitative role of sanctions does not appear to have been the focus of a theoretical study. However, Isaac Ehrlich (1981) contains an interesting section on incapacitation which emphasizes so-called replacement effects; there is little overlap between his paper and this one.

[3] This may be interpreted as including not only the costs associated with the building and operation of prisons, but also the forgone production of the imprisoned individual and the disutility he suffers.

The social problem is to choose a system of sentences and the probability of apprehension to minimize expected social costs, defined as the expected sum of harm, the costs of imprisonment, and enforcement expenses.

The optimal system of sentences is clear: an apprehended individual for whom $h > c$ will be imprisoned for life, but one for whom $h \leqq c$ will be set free. This is because individuals for whom $h > c$ do more harm any period that they are free than they cost to imprison, and conversely if $h \leqq c$. (If $h = c$, it does not matter whether individuals go free, but for simplicity I adopt the convention that they do.)

Note that the optimal sentences do not depend on p or on n, but they do depend on c.

Given that sentences are optimal, social costs per period as a function of the probability of apprehension are

$$(1)\quad n\int_0^c hf(h)\,dh + q(p)\int_c^{\bar{h}} hf(h)\,dh + (n - q(p))\int_c^{\bar{h}} cf(h)\,dh + e(p),$$

where

$$(2)\quad q(p) = (1-p) + (1-p)^2 + \ldots + (1-p)^n.$$

That is, the sum of the probabilities that individuals in different age cohorts have not been apprehended, and thus where the sum of the probabilities of individuals in the different cohorts who have been apprehended is

$$(3)\quad [1-(1-p)] + \ldots + [1-(1-p)^n] = n - q(p).$$

The first-order condition determining the optimal p is therefore

$$(4)\quad e'(p) = -q'(p)\int_c^{\bar{h}} (h-c)f(h)\,dh.$$

Namely, marginal enforcement expenses must equal the reduction in harm (net of the cost of imprisonment) due to imprisonment of additional individuals.

It follows from (4) that the optimal probability rises with n; since $-q'(p) = 1 + 2(1-p) + \ldots + n(1-p)^{n-1}$ rises with n, p must rise to maintain equality in (4). (If individuals do harm for a longer time, it is more important to incapacitate them.) Also the optimal probability rises with a rightward shift in the distribution of h; if the harm done by a person of type h is $h + t$, where t is a parameter, then the right-hand side of (4) becomes $-q'(p)\int_{c-t}^{\bar{h}}(h+t-c)f(h)\,dh$. Differentiating this with respect to t gives $-q'(p)\times$(Probability that h exceeds $c - t$), which is positive. Hence, again, p must rise to maintain equality in (4). In addition the optimal probability rises with a decline in c; differentiating the right-hand side of (4) with respect to c gives $q'(p)\times$ (Probability that h exceeds c), which is negative. (Since a decline in c means it is optimal to imprison a greater percentage of individuals who are apprehended, the social payoff from raising the probability is enhanced.)

B. *Harmfulness a Function of Age*

Suppose now that the harm an individual will do if free declines with his age. (There is strong evidence that dangerousness does decline with age; see, for instance, U.S. Department of Justice.) Then it is clear that it will be optimal to imprison an apprehended individual for whom $h > c$ and to release him as soon as $h \leqq c$. The optimal probability of apprehension will be determined by a condition similar to (4) (and will fall) but for simplicity the probability will not be reconsidered here or below.

C. *Rehabilitative Effect of Imprisonment*

Suppose next that time spent in prison will reduce an individual's harmfulness (and, to isolate this rehabilitative effect, suppose that age itself will not reduce harmfulness). In particular, assume that each period an individual is in prison, his harmfulness will be multiplied by a factor r, where $1 > r > 0$; hence if he is in prison i periods and released, the harm he will do each subsequent period will be $r^i h$. It will be shown that the optimal sentencing policy depends on a comparison with the threshold

$$(5) \qquad t(j)=c/[j-(j-1)r],$$

where j is the number of periods remaining in an individual's life. Thus $t(1)=c>t(2)=c/(2-r)>t(3)=c/(3-2r)\ldots$. The optimal sentencing policy is to imprison an apprehended individual with j periods left in his life if $h>t(j)$, and to release him if, as may happen, his harmfulness later falls to or below the then-relevant threshold.

Notice that since $t(j)<c$ for $j>1$, it may be optimal to imprison an individual even though his current harmfulness h is less than the per period cost of imprisonment c. The reason is that the rehabilitative effect of imprisonment means that when he is later freed, he will do less harm; thus imprisonment will benefit society in a way in addition to incapacitating the individual. The rehabilitative effect is more important if an individual is young since society will have more time in which to accomplish rehabilitation and more time in which to enjoy its benefits; thus the threshold rises with age. The rehabilitative effect is of no value to society if an individual has only one period left to live, so it makes sense that $t(1)=c$.

That the optimal sentencing policy is as claimed can be proved by induction. For $j=1$, it is obvious that it is optimal to imprison an individual if $h>c$ and not otherwise. Consider now an apprehended individual with $j>1$ periods of life remaining for whom $h<t(j-1)$. We know by the inductive hypothesis that whether or not the individual is imprisoned this period, it will be optimal for him to be free for all periods after this period (since $h<t(j-1)$). Hence, if he is allowed to go free this period, social costs associated with him will be jh, and if he is imprisoned this period, the social costs will be $c+(j-1)rh$. Accordingly, as long as $jh\leqq c+rh(j-1)$, it will be optimal to allow him to go free. Solving $jh=c+rh(j-1)$ for h, we obtain the asserted threshold $c/[j-(j-1)r]=t(j)$. (It was assumed here that $h<t(j-1)$; if h is larger, the argument that it continues to be optimal to imprison an individual is tedious and is omitted.)

D. *Death Penalty vs. Imprisonment*

Suppose last that the situation is as in the basic model except that the death penalty is available as an alternative to imprisonment, where the death penalty involves a social cost d.[4] We know that, in the absence of the death penalty, it is optimal to imprison an individual for life if $h>c$ and to allow him to go free otherwise. This means that, in the absence of the death penalty, for an individual who is apprehended and who will live j more periods, social costs will be augmented by jc if $h>c$ and jh otherwise. Hence, it will be optimal to apply the death penalty if and only if $d<\min(jh, jc)$. Thus, for given j, the death penalty will not be employed for any h if $d\geqq jc$; and if $d<jc$, the penalty will be imposed if $h>d/j$ (and individuals will be set free otherwise). The penalty is more likely to be optimal when j is large since then society will save more in imprisonment costs.

II. Concluding Comment: Optimal Sanctions when Incapacitation is the Goal vs. when Deterrence is the Goal

There are several important differences between the optimal magnitude of sanctions when incapacitation is the goal from when deterrence is the goal.[5] First, when deterrence is the goal, the optimal sanction is generally higher the lower is the probability of apprehending an individual. But as seen here, when incapacitation is the goal, the optimal sanction is independent of the probability of apprehension (although the optimal probability depends on the optimal

[4]The cost d could be interpreted as including not only the resource costs of imposing the death penalty, which might be small, but also an amount reflecting social distaste for the penalty or its disadvantageous effects on the public perception of the value of human life. Note also that the sanction of permanent deportation is, for our purposes, qualitatively identical to the death penalty, since it removes an individual from the population and involves (for the deporting country) a once and for all cost.

[5]See Gary Becker's original paper (1968) on the use of sanctions to deter, and see also R. Carr-Hill and Nicholas Stern (1979), A. Mitchell Polinsky and myself (1984), and my 1985 paper.

sanctioning policy). Second, when deterrence is the goal, the optimal sanction generally rises continuously with the magnitude of the harm done, but when incapacitation is the goal, the sanction rises discontinuously, from zero to a positive and fixed level once a threshold level of harm is surpassed. Third, when deterrence is the goal, the optimal magnitude of sanctions depends on the ability to deter, and if this ability is small (as, for instance, with the enraged), a low optimal sanction will be indicated. But a high sanction might be called for to incapacitate.

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