Specific versus General Enforcement of Law

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Optimal enforcement of law is examined in a model with specific enforcement effort—effort devoted toward apprehending individuals who have committed a single type of harmful act—and general enforcement effort—effort devoted toward apprehending individuals who have committed any of a range of harmful acts (a police officer on patrol, for instance, is able to apprehend many types of violators of law). If enforcement effort is specific, optimal sanctions are extreme for all acts. If enforcement effort is general, however, optimal sanctions rise with the harmfulness of acts and reach the extreme only for the most harmful acts.

The problem of optimal public enforcement of law—the problem of selecting probabilities and magnitudes of sanctions that best deter violations—is examined here in a model in which two types of enforcement effort, specific and general, are distinguished.

By *specific* enforcement effort, I mean activity devoted to apprehending and penalizing individuals who have committed a *single* type of harmful act, identified by its degree of harmfulness, among its other characteristics. The activity of an employee of a traffic department whose sole duty is to ticket people for overtime parking exemplifies specific enforcement effort. So does, typically, investigative or prosecutorial effort made *after* the commission of a harmful act, for such effort by its nature concerns a single act.

In contrast, *general* enforcement effort is activity affecting the likelihood of apprehension of individuals who have committed any of a

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range of harmful acts. A police officer on patrol, for instance, is able to apprehend many types of violators of law, from those who shoplift, to those who engage in assault, to those who commit murder; an Internal Revenue Service agent screening tax returns may learn about multiple kinds of errors and false claims, such as overstating charitable contributions or understating nonwage income, and each of these may vary in amount. Whenever an enforcement agent's activity involves monitoring that naturally allows him to detect different types of violators, or violators who do different amounts of harm, enforcement activity is what is called here general.[1]

Of course, many enforcement activities have both specific and general aspects. When an IRS agent screens only charitable deductions, his activity has a specific feature, but, presumably, the amount of the error or the false claim may vary, so that his activity also has a general dimension. It will be clear to readers how to interpret such mixed cases once they appreciate the significance of the distinction between the two paradigms of enforcement activity with which we shall be concerned in the analysis.

To that end, consider initially the assumption that *all enforcement effort is specific*. This means that the enforcement of law concerning one harmful act is independent of the enforcement concerning any other; society may devote one level of specific enforcement effort toward apprehension of individuals who commit one act (and set one sanction for it) and may devote a very different level of specific enforcement effort toward apprehension of those who commit another act. This implies, under wide assumptions, that it is optimal for sanctions to be extreme, as high as possible,[2] for all acts.

The reasoning is well known and is due essentially to Becker (1968). To review, if the sanction for an act is not extreme, society should enjoy an opportunity to conserve enforcement resources without sacrificing deterrence: Society should be able to reduce enforcement effort and to augment the (less than extreme) sanction by an amount calculated to leave the expected sanction—and thus deterrence of the act—unchanged. At the optimum, it must be impossible for society to use this beneficial stratagem involving an increase in the sanction; that is, it is optimal for the sanction to be extreme.[3] Because this

[1] The term "general enforcement" should not be confused with "general deterrence," which refers to the tendency of the threat of punishment to dissuade people generally from committing bad acts. Nor should "specific enforcement" be confused with "specific deterrence" or with "particular deterrence," which often are taken to refer to the tendency of punishment of a particular individual to induce him not to commit bad acts in the future. On the terms particular deterrence and general deterrence, see, e.g., LaFave and Scott (1972, pp. 22–23) and the articles cited therein.

[2] It is assumed that sanctions have some bound (see n. 8).

[3] While Becker was the first to notice that society may have a beneficial opportunity

argument applies independently to each act, it appears that the optimal sanction for each act is extreme.

In the analysis below this conclusion is considered formally and is verified to be correct when the sanction is solely monetary or is solely imprisonment.[4] When sanctions are combined, the conclusion is modified somewhat; only the monetary component need be extreme.

However, the conclusion that sanctions should tend toward the extreme is at odds with what is observed in fact. Extreme sanctions are not the norm but the exception.

A conclusion about optimal sanctions more in accord with what is observed is reached when one takes into account general enforcement effort. Assume for simplicity, as is done in part of the analysis, that *all enforcement effort is general* and that enforcement effort results in the same probability of apprehension for all acts. Now to deter reasonably well the totality of harmful acts, a certain probability of apprehension will be required. Because this probability of apprehension will apply in particular to those who commit less harmful acts, the probability will be more than sufficient to deter these acts appropriately if extreme sanctions are employed, so that extreme sanctions will not be needed.[5]

This point may be restated less abstractly. Society wants a certain number of police on the streets to deter the whole range of crimes, including, especially, serious ones. But given that these police are on the streets, they will be present to apprehend those who commit lesser crimes. Society therefore does not need to threaten those who would commit lesser crimes with the very high sanctions it employs for serious crimes.

More precisely, what will be shown in the analysis in which enforcement effort is general is that optimal sanctions are low for acts of small harmfulness, increase with the degree of harmfulness, and

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to reduce enforcement effort and increase sanctions, he did not stress that this point leads to the conclusion that extreme sanctions are optimal, and he proceeded for the most part as though less than extreme sanctions are optimal. That his argument implies that extreme sanctions are optimal has, however, been noted by others (see, e.g., Carr-Hill and Stern 1979, pp. 280–309; Polinsky and Shavell 1979).

[4] The precise result shown is that optimal sanctions are extreme if they are positive; however, optimal sanctions are zero for all acts resulting in harm below a certain threshold.

[5] Another theoretical justification for less than extreme sanctions involves risk aversion on the part of sanctioned parties (see Polinsky and Shavell 1979; Kaplow 1989). In the present paper, individuals are assumed to be risk neutral. An additional justification for less than extreme sanctions concerns "marginal deterrence," inducing the undeterred to commit less harmful rather than more harmful acts by setting a lower sanction for less harmful acts than for more harmful acts (see Stigler 1970). This justification, however, actually depends implicitly on an assumption of general enforcement effort (see Reinganum and Wilde 1986; Wilde 1989; Shavell, in press).

reach the extreme only for the most harmful acts.[6] This is true whether the form of sanction is solely monetary, is solely imprisonment, or may be a combination of the two. In the last case, optimal sanctions are at first purely monetary and rise with the degree of harm to the highest level, an individual's wealth; then these extreme monetary sanctions are accompanied by imprisonment that increases with the level of harm. This result, it may be remarked, is in rough accord with reality in that criminal sanctions are reserved for seriously harmful acts and increase with the gravity of the acts.

Also considered in the analysis is the assumption that *enforcement effort may be both general and specific*. Under this assumption, the conclusions are similar to those just discussed in one sense: it is optimal for less than extreme sanctions to be used for all but the most harmful acts; the reason is again that since there is general enforcement effort, the probability of apprehending those who commit less harmful acts is more than enough to deter adequately if extreme sanctions are employed. However, society is able to exercise specific enforcement effort as well, and it will be worth society's while for specific enforcement effort to augment general enforcement effort for acts that are sufficiently harmful. Thus, for instance, it will be worth society's while to devote police resources to investigate acts if their harmfulness exceeds a threshold (a theft of, say, more than \$100), but otherwise to rely on general enforcement activity to apprehend violators.[7]

The paper closes with several comments on possible extensions of the analysis.

I. The Model

Risk-neutral individuals decide whether to commit harmful acts. Individuals differ; a particular type of individual is identified by the benefit he would obtain from his act and by its harmfulness. Define b as the benefit from committing an act ($b \geq 0$); $f(b)$ as the probability

[6] A numerical example illustrates this conclusion. Suppose that there are two types of acts, those causing harm of \$1 and those causing harm of \$100. To deter properly the more harmful acts, the expected sanction should equal \$100. Following Becker, suppose that this is done as cheaply as possible, by using the extreme sanction of a person's entire wealth—say it is \$10,000—and along with it a low probability of apprehension—here a probability of only 1 percent (for $.01 \times \$10{,}000 = \$100$). Because the probability of 1 percent is general and applies also to those who commit the act causing only the \$1 harm, a sanction of just \$100 will be optimal for that act (as $.01 \times \$100 = \$1$); in other words, a sanction far less than a person's entire wealth will be optimal.

[7] Mookherjee and Png (1989) obtain a similar result in a model considering the optimal joint use of monitoring effort and of investigation of reported violations (their paper and the present one were written independently of each other). See n. 13 below.

density of b; f is continuous, bounded, and positive on $[0, \infty)$; h as the harm due to an act ($h \geq 0$); and $g(h)$ as the probability density of h; g is continuous, bounded, and positive on $[0, \infty)$. The distribution of benefits is assumed for simplicity to be the same for different h.

If an individual commits a harmful act, he will suffer a sanction with a probability. The sanction may be solely monetary, may be solely imprisonment, or may be a combination of the two. Let $s(h)$ be the monetary sanction for committing an act causing harm h, $z(h)$ the imprisonment for committing an act causing harm h, w the wealth of individuals, and p the probability of apprehension; p may or may not depend on h, as specified. It is assumed that the social authority imposing sanctions can observe h; thus the sanctions can be made a function of h. A monetary sanction cannot be higher than an individual's wealth, which is assumed to be equal for all individuals (but see the comment on this assumption in the concluding section). Hence,

$$0 \leq s(h) \leq w. \tag{1}$$

It is assumed also that imprisonment is bounded by some maximal sanction $\bar{z}$. This is justified by the usual axioms of expected utility theory; they imply that utility, or disutility, is bounded.[8] Hence,

$$0 \leq z(h) \leq \bar{z}. \tag{2}$$

If an individual suffers imprisonment, it is assumed that society bears a cost; let σz be the social cost if imprisonment z is imposed ($\sigma > 0$). That imposition of imprisonment is assumed socially costly is motivated by two considerations. First, the disutility suffered by an individual may be considered a social cost.[9] Second, imposition of imprisonment involves resource costs (the expenses of operating the prison system).

Because individuals are risk neutral, an individual will commit an act if and only if his benefit is at least as large as the expected sanction,[10]

$$b \geq p[s(h) + z(h)]. \tag{3}$$

The probability of apprehension p is determined by enforcement

[8] See, e.g., Arrow (1971, pp. 63–69). Block and Lind (1975) emphasize the boundedness of utility in an early discussion of the use of sanctions.

[9] Note by contrast that the imposition of a monetary sanction is not natural to consider as a social cost, for what the penalized party pays someone else receives; imposition of monetary sanctions involves only a transfer of command over resources. Imposition of imprisonment creates a disutility that is not balanced in any automatic way by an increase in the utility of another.

[10] I assume for concreteness that if there is equality in (3), the individual will commit the act even though he is indifferent between doing so and not doing so.

effort, of which, as explained above, there are two types, specific and general. Specific enforcement effort raises the probability of apprehension for those who commit a specific type of harmful act, identified by h. General enforcement effort raises the probability of apprehension of all individuals who commit harmful acts, whatever h is. Let $x(h)$ be the enforcement effort specific to apprehending those who commit acts causing harm h and y the general enforcement effort. As stated in the Introduction, three cases will be studied. In the first, all enforcement effort is specific; here it is assumed that, for any h,

$$p = p(x(h)), \tag{4}$$

where p and its derivatives are defined for $x \geq 0$, $p(0) = 0$, $0 \leq p(x) < 1$, $p'(x) > 0$, and $p''(x) < 0$; that is, the probability of apprehending any given type of individual is zero if no effort is made and increases with enforcement effort, but at a decreasing rate. (In the concluding section the assumption implicitly made here that the probability is the same function of x for all h is briefly discussed.) Total specific enforcement effort is

$$\int_0^\infty x(h)dh. \tag{5}$$

In the second case, enforcement effort is general. In this case, for all h,

$$p = p(y), \tag{6}$$

where p has the same properties as before, and total enforcement effort is y. In the third case, enforcement effort is both specific and general, and, for any h,

$$p = p(x(h), y), \tag{7}$$

where p is increasing and concave in x and y, and total enforcement effort is given by expression (5) plus y.

Social welfare is defined to be the benefits individuals obtain from committing acts, less the harm done, less the social costs of imposing imprisonment, less total enforcement effort.

I shall now consider the problem of choosing sanctions and enforcement effort—and thus the probability of apprehension—so as to maximize social welfare in the three cases. The three cases will be examined first for a sanction that is solely monetary, then for one that is solely imprisonment, and finally for one that is a combination of both. This will allow us to build a fairly complete understanding of the solution to the enforcement problem.

A. *Sanctions Are Solely Monetary*

If enforcement effort is specific, the social problem is a set of entirely independent problems; for each h, enforcement effort and a sanction must be optimally selected. Social welfare is given by

$$\int_0^\infty \int_{p(x(h))s(h)}^\infty (b - h)f(b)dbg(h)dh - \int_0^\infty x(h)dh; \tag{8}$$

the social problem is to maximize (8) over functions $x(h)$ and $s(h)$. Equivalently, the social problem is to choose, for each h, enforcement effort x and a sanction s to maximize

$$\int_{p(x)s}^\infty (b - h)f(b)dbg(h) - x. \tag{9}$$

The solutions to this problem will be denoted x^* (or $x^*(h)$) and s^* (or $s^*(h)$), and an asterisk will generally denote optimal values below. The following result holds.[11]

PROPOSITION 1. Suppose that enforcement effort is specific and that monetary sanctions alone are employed. Then for all harms h below a threshold, optimal enforcement effort is zero. Where optimal enforcement effort is positive, the optimal sanction is maximal, equal to wealth.

Remarks.—The proposition is illustrated in figure 1. That it is not worthwhile expending enforcement effort for small harms is readily explained: the marginal cost of effort is one, but the social benefit due to deterrence of harms tends to zero as the harms tend to zero. That the sanction should always equal wealth when enforcement effort is positive[12] is due to Becker's argument: if the sanction were less than wealth, it could be raised and enforcement effort lowered so as to save resources but maintain deterrence.

When enforcement effort is general, the social problem is no longer a set of independent problems, one for each h. Instead, the enforcement problems for different h are interconnected because a single probability of apprehension applies for all h. The social problem is to choose general enforcement effort y and sanctions $s(h)$ to maximize social welfare:

$$\int_0^\infty \int_{p(y)s(h)}^\infty (b - h)f(b)dbg(h)dh - y. \tag{10}$$

I assume for simplicity that optimal general enforcement effort y^* is positive; the probability $p(y^*)$ will be denoted p^*. The following result obtains.

[11] The proofs to this and other propositions appear in the Appendix.

[12] When enforcement effort $x^*(h)$ is positive, it is shown as rising with h in fig. 1, but this need not be the case (nor need it be the case in fig. 3 below).

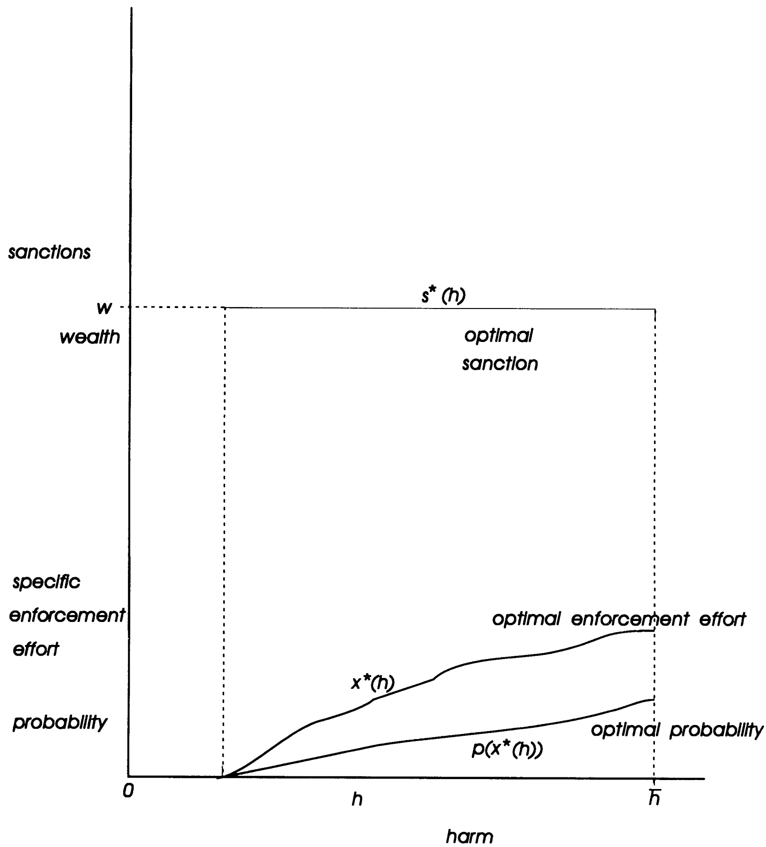

FIG. 1.—Specific enforcement effort with monetary sanctions

PROPOSITION 2. Suppose that enforcement effort is general and that monetary sanctions alone are employed. Then for all harms h below the threshold p^*w, the optimal sanction is given by the formula h/p^*; the expected sanction thus equals the harm h and rises with the level of harm. For harms above the threshold, the optimal sanction is maximal, equal to wealth, and there is underdeterrence.

Remarks.—The proposition is illustrated in figure 2. Because enforcement effort is general and one probability of apprehension applies for all h, the probability is high enough to allow achievement of perfect deterrence for h below a threshold. This threshold is at the point at which the maximum expected sanction p^*w equals the harm.

When both general enforcement effort and specific enforcement effort may be employed, the social problem is to choose specific enforcement effort $x(h)$, general enforcement effort y, and sanctions

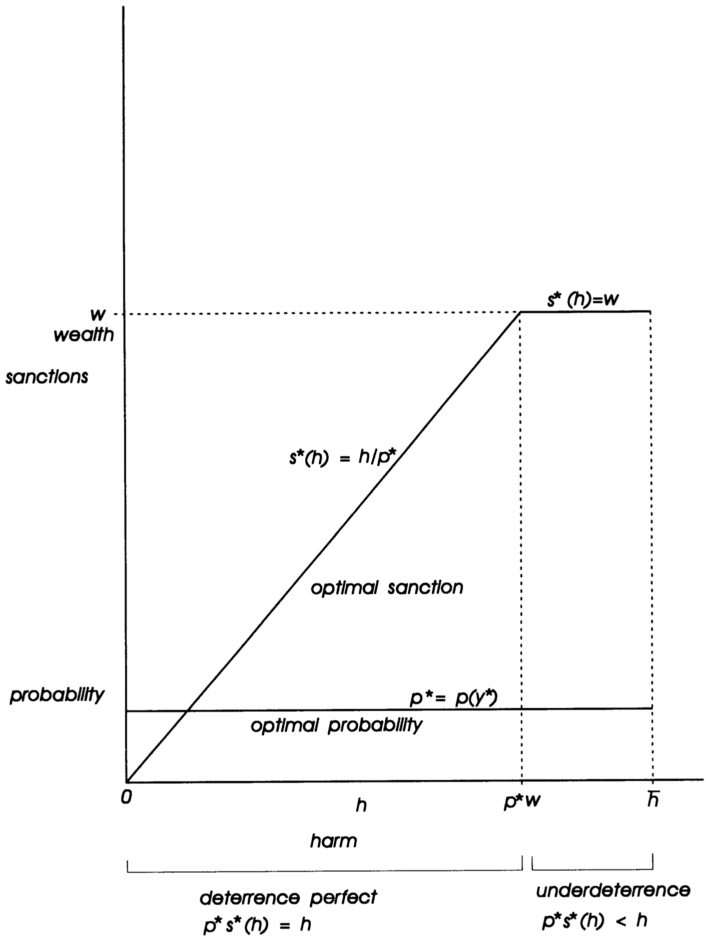

FIG. 2.—General enforcement effort with monetary sanctions

$s(h)$ to maximize social welfare:

$$\int_0^{\infty}\int_{p(x(h),y)s(h)}^{\infty}(b - h)f(b)dbg(h)dh - \int_0^{\infty}x(h)dh - y. \tag{11}$$

I assume that optimal general enforcement effort y^* is positive and that $x^*(h)$ is positive for some h (otherwise the social problem devolves into one of the two problems that have already been considered), and we have the following proposition.

PROPOSITION 3. Suppose that general enforcement effort may be augmented by specific effort and that monetary sanctions alone are employed. Then for all harms below the threshold p^*w, the optimal sanction equals h/p^*—hence the expected sanction equals the harm

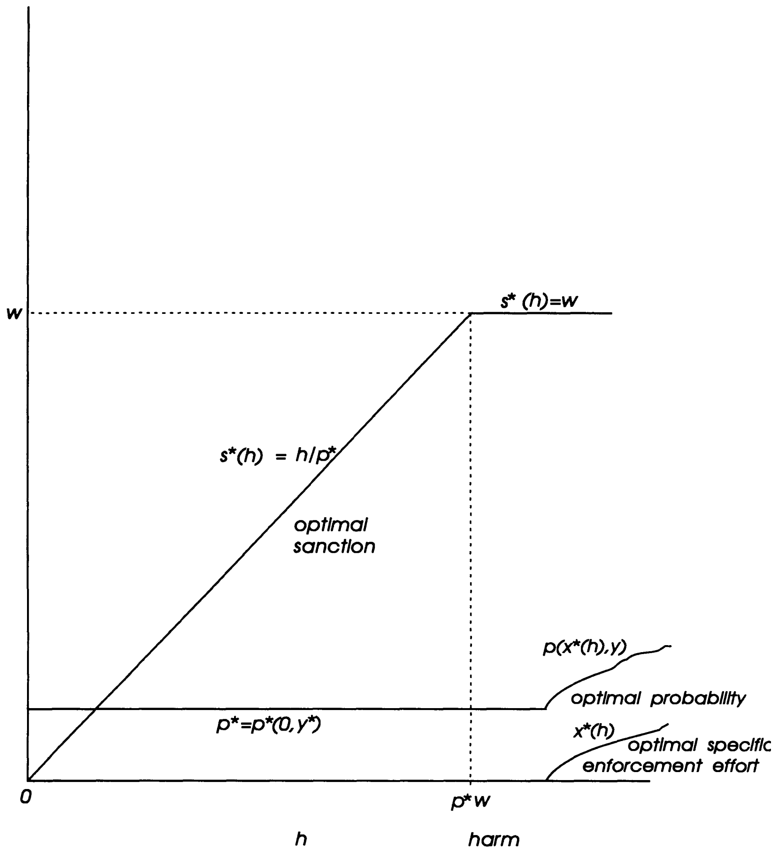

FIG. 3.—General and specific enforcement effort with monetary sanctions

h and rises with the level of harm—and optimal specific enforcement effort is zero. Beyond the threshold p^*w, the optimal sanction equals the maximal level, wealth, and positive specific effort becomes optimal at a level of harm strictly greater than the threshold.

Remarks.—The proposition is illustrated in figure 3. The explanation for the results is that in the first region of figure 3, perfect deterrence is possible without supplementing general enforcement effort with specific enforcement effort. After the sanction becomes maximal, there is a problem of underdeterrence; when this problem becomes important enough, specific enforcement effort is worthwhile.[13]

[13] The explanation for a similar result of Mookherjee and Png (1989, proposition 3) is related, although their model is different from the present one. Notably, in their model, each individual chooses from among a continuum of possible acts.

B. Sanction Is Solely Imprisonment

In this case, the results and proofs are in most respects similar to those in which sanctions were monetary. If enforcement effort is specific, social welfare is

$$\int_0^\infty \int_{p(x(h))z(h)}^\infty [b - h - \sigma p(x(h))z(h)]f(b)dbg(h)dh - \int_0^\infty x(h)dh; \quad (12)$$

the social problem is to maximize (12) over functions $x(h)$ and $z(h)$. We have the following proposition.

PROPOSITION 4. Suppose that enforcement effort is specific and that imprisonment alone is employed. Then for all harms below a threshold, optimal enforcement effort is zero. Where optimal enforcement effort is positive, the optimal sanction is always maximal, equal to $\bar{z}$.

Remarks.—The explanation for this result is like that for proposition 1. It should be noted that Becker's argument that the optimal sanction is maximal still applies. In particular, if the sanction is not maximal and is raised and enforcement effort is reduced so that the expected sanction is not altered, then not only is deterrence maintained, but also the expected social cost of imposing sanctions is left unchanged.

Indeed, it is straightforward to verify that the conclusion that optimal imprisonment is maximal continues to hold if the marginal disutility of imprisonment increases with its magnitude (imprisonment becomes harder as time passes) or if the marginal social cost of imposing imprisonment decreases. On the other hand, the conclusion does not necessarily hold if the marginal disutility of imprisonment decreases or if the marginal social cost of imprisonment increases.

The other two results are as follows.[14]

PROPOSITION 5. Suppose that enforcement effort is general and that imprisonment alone is employed. Then for all harms below a threshold, the optimal sanction is zero; above this threshold, optimal sanctions are positive and rise with the level of harm, attaining the maximal level, $\bar{z}$, for all harms beyond some point.

PROPOSITION 6. Suppose that general enforcement effort may be augmented by specific enforcement effort and that imprisonment alone is the sanction. Then the optimal sanction is at first zero and subsequently rises with harm until it equals the maximal amount $\bar{z}$. Optimal specific enforcement effort is zero until sanctions become maximal, after which optimal specific enforcement effort becomes positive.

[14] Shavell (1987) also analyzes general enforcement effort and the sanction of imprisonment, but emphasizes the importance of the social authority's information about b.

C. Sanctions Are Combined

If both monetary sanctions and imprisonment may be used and enforcement is specific, the social problem is to choose $x(h)$, $s(h)$, and $z(h)$ to maximize

$$\int_0^\infty \int_{p(x(h))[s(h)+z(h)]}^\infty [b - h - \sigma p(x(h))z(h)]f(b)db g(h)dh - \int_0^\infty x(h)dh, \qquad (13)$$

and we have the following proposition.

PROPOSITION 7. Suppose that enforcement effort is specific and that both monetary sanctions and imprisonment may be employed. Then for all harms h below a threshold, optimal enforcement effort is zero. Where optimal enforcement effort is positive, the optimal monetary sanction is always maximal, equal to wealth, but the imprisonment sanction may not be maximal.

Remarks.—It is worth discussing why the imprisonment sanction may not be maximal (and could be zero) when enforcement effort is positive. By now familiar logic, the optimal monetary sanction equals wealth w. This, however, means that Becker's argument does not necessarily apply to imprisonment. Specifically, suppose that the imprisonment z is less than maximal, and raise z slightly and lower enforcement effort so that the expected sanction is held constant. But when enforcement effort and the likelihood of apprehension p are lowered, the likelihood of imposing the monetary sanction w is lowered. This means that to maintain the level of the expected sanction, p cannot be reduced in proportion to the increase in z; p must be reduced less than proportionately. This implies that the social cost of imposing imprisonment rises, so that it is not clear that social welfare rises.

If enforcement effort is general, the social problem is to choose y, $s(h)$, and $z(h)$ to maximize

$$\int_0^\infty \int_{p(y)[s(h)+z(h)]}^\infty [b - h - \sigma p(y)z(h)]f(b)db g(h)dh - y, \qquad (14)$$

and assuming that y^* is positive, we shall show the following proposition.

PROPOSITION 8. Suppose that enforcement effort is general and that both monetary sanctions and imprisonment may be employed. Then for all harms below the threshold p^*w, the optimal sanction is purely monetary and equals h/p^*; the expected sanction thus equals the harm h and rises with harm. Above the threshold, the optimal monetary sanction is maximal, equal to wealth w, and optimal imprisonment is at first zero and then becomes positive.

Remarks.—The proposition is illustrated in figure 4. The reason that purely monetary sanctions are initially employed is that it is

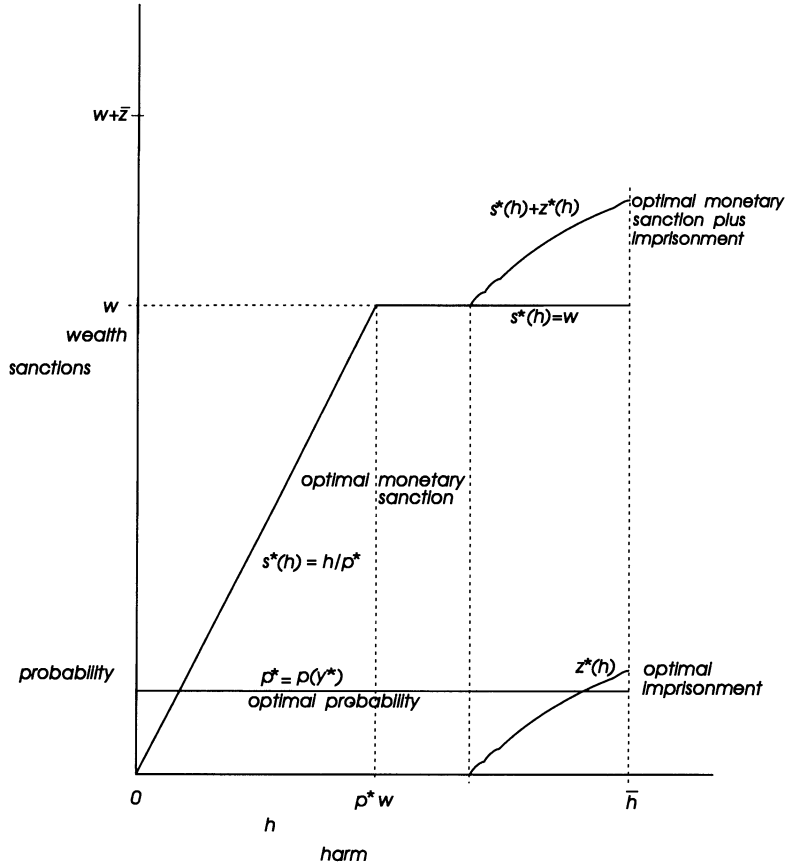

FIG. 4.—General enforcement effort with monetary sanctions and imprisonment

wasteful to impose socially costly imprisonment when socially costless monetary sanctions can be used in their place. However, beyond the threshold p^*w, the wealth constraint on monetary sanctions implies that there is underdeterrence, and it thus becomes desirable to employ imprisonment as well.

If enforcement effort is both general and specific, the social problem is to choose $x(h)$, y, $s(h)$, and $z(h)$ to maximize social welfare:

$$\int_0^\infty \int_{p(x(h),y)[s(h)+z(h)]}^\infty [b - h - \sigma p(x(h),y)z(h)]f(b)db\,g(h)dh - \int_0^\infty x(h)dh - y, \tag{15}$$

and, assuming as before that y^* is positive and that $x^*(h)$ is positive for some h, we have the following proposition.

PROPOSITION 9. Suppose that general enforcement effort may be augmented by specific effort and that monetary sanctions and imprisonment may be employed. Then below the threshold p^*w, the optimal sanction is purely monetary and equals h/p^* so that the expected sanction equals the harm h and rises with the level of harm; also, optimal specific enforcement effort is zero. Beyond h/p^*, the optimal monetary sanction is maximal, wealth, and optimal imprisonment and specific enforcement effort eventually become positive.

II. Conclusion

It is worthwhile indicating how relaxation of several of the assumptions of the model would alter the conclusions. One assumption was that all individuals have the same wealth. Were wealth allowed to vary among individuals, then, presumably, it would become optimal to impose imprisonment on those who, because of their inadequate wealth, could not pay an otherwise optimal solely monetary sanction. In other words, for certain violations, relatively wealthy individuals would bear only monetary sanctions, whereas other individuals would suffer imprisonment as well (and the lower their wealth, the higher the imprisonment).

Another assumption was that general enforcement effort resulted in the same probability of apprehension for those committing different harmful acts. This assumption could be altered to allow for the effect of general enforcement effort to vary according to the act. We know in fact that when a police officer is on patrol, the likelihood of his apprehending different types of violators is different; the chance of his catching a burglar may be lower than the chance of his catching a person who commits an assault. In formal terms, the probability of apprehension p could be a function not only of general enforcement effort y but also of the type of act h, that is, $p = p(y, h)$. Were this the assumption, the formula for (less than extreme) optimal sanctions would be $h/p(y^*, h)$ rather than $h/p(y^*)$. Hence, it might not be the case that sanctions rise with harm, for if p happens to rise with h over some range, optimal sanctions might fall.

A similar assumption was that the probability of apprehension was the same function of specific enforcement effort for each type of harmful act. Were this assumption altered, the conclusions would change in obvious ways; for instance, optimal specific enforcement effort would tend to be higher than we found in the case in which such effort would be very productive in raising the probability.

Appendix

Proof of Proposition 1

The argument consists of several steps.

i) If $x^*(h) > 0$, then $s^*(h) = w$: Assume otherwise, that $x^* > 0$ and $s^* < w$.

Raise s to w and lower x to x' such that $p(x')w = p(x^*)s^*$. (This is obviously possible.) Then the integral in (9) remains the same—the same individuals commit the act—but enforcement effort is lower. Thus (9) is higher, which contradicts the assumption that x^* and s^* were optimal.

ii) $x^*(h) = 0$ for all h sufficiently low: From part i, we know that if x^* is positive, $s^* = w$, so that x^* in fact maximizes

$$\int_{p(x)w}^{\infty} (b - h)f(b)dbg(h) - x \qquad \text{(A1)}$$

over x. Differentiating (A1) with respect to x, we obtain

$$-p'(x)w(pw - h)f(pw)g(h) - 1. \qquad \text{(A2)}$$

The first term in (A2) is the marginal gain due to increased deterrence ($h - pw$ is the net social loss avoided when the marginal individual is deterred) and 1 is the marginal cost of raising enforcement effort. If (A2) is negative for all x, then $x^* = 0$, and (A2) is in fact negative for all x if h is sufficiently small.[15] Q.E.D.

Notes

a) If h is high enough so that x^* is positive, then x^* is determined by

$$-p'(x)w(pw - h)f(pw)g(h) - 1 = 0. \qquad \text{(A3)}$$

The sign of $x^{*\prime}(h)$ equals the sign of the partial derivative of the left-hand side of (A3) with respect to h,[16] which can be positive or negative.

b) If $x^*(h) > 0$, the expected sanction $p(x^*(h))w$ is less than h. This is apparent from (A3).

Proof of Proposition 2

$s^*(h) = h/p^*$—so that $p^*s^*(h) = h$—for $h < p^*w$; $s^*(h) = w$ for larger h: Given y and p, the social problem for any h is to maximize over s

$$\int_{ps}^{\infty} (b - h)f(b)db, \qquad \text{(A4)}$$

the derivative of which with respect to s is

$$-p(ps - h)f(ps). \qquad \text{(A5)}$$

This is positive when $ps < h$ or when $s < h/p$, it is zero at $s = h/p$, and it is negative for larger s. It follows that (A4) is maximized at $s = h/p$ if this s is feasible; that is, $s^*(h) = h/p^*$ if $h/p^* \leq w$ or if $h \leq p^*w$. Otherwise, $s^*(h) = w$ since (A5) is positive when $p^*s < h$. Q.E.D.

[15] The first term of (A2) equals $-p'(x)pw^2f(pw)g(h) + hp'(x)wf(pw)g(h)$. Now $hp'(x)wf(pw)g(h)$, which is positive, is bounded over all x by $hp'(0)wf_bg(h)$, where f_b is a bound for the density f. Hence, for all h sufficiently small, $hp'(x)wf(pw)g(h)$ is dominated by -1, so (A2) is indeed negative for all x for such h.

[16] The condition (A3) has the form $W(x, h) = 0$. Implicitly differentiating with respect to h, one obtains $W_x x^{*\prime} + W_h = 0$, so that $x^{*\prime} = -W_h/W_x$. But $W_x < 0$; this is the second-order condition for x^* to be a maximum. Hence, the sign of $x^{*\prime}$ is the sign of W_h.

Notes

It follows from the proof that social welfare (10) may be written as

$$\int_0^{p(y)w}\int_h^{\infty}(b-h)f(b)dbg(h)dh+\int_{p(y)w}^{\infty}\int_{p(y)w}^{\infty}(b-h)f(b)dbg(h)dh-y. \quad \text{(A6)}$$

The first term is associated with the region of h over which deterrence is perfect, since $ps(h) = ph/p = h$; the second term is associated with the region of h over which $s(h) = w$ and there is underdeterrence. Differentiating (A6) with respect to y, we obtain the first-order condition

$$p'(y)wf(p(y)w)\int_{p(y)w}^{\infty}[h-p(y)w]g(h)dh = 1, \quad \text{(A7)}$$

determining y^* and p^*.[17] The left-hand side is the marginal benefit from increasing y and p, which inheres in reducing social losses by $h - p(y)w$ for persons just deterred in the region of h above $p(y)w$.

Proof of Proposition 3

i) If $x^*(h) > 0$, then $s^*(h) = w$: The social problem for any h is to choose s and x to maximize

$$\int_{p(x,y)s}^{\infty}(b-h)f(b)dbg(h)-x. \quad \text{(A8)}$$

If $x^* > 0$ but $s^* < w$, raise s to w and lower x to x' such that $p(x', y)w = p(x^*, y)s^*$. Then the integral in (A8) remains the same; but since $x' < x^*$, (A8) is higher, a contradiction.

ii) If $h < p^*w$, then $s^*(h) = h/p^*$ and $x^*(h) = 0$, where $p^* = p(0, y^*)$: Assume first that $s^* < w$. Then by part i, $x^* = 0$. This means that maximization of (A8) reduces to maximization of (A4), which we know is maximized at $s^* = h/p^*$ (for $h/p^* < w$ since $h < p^*w$), where $p^*s^* = h$. Now assume that $s^* = w$. In this case, however, social welfare is lower: Because $p^*s^* = p^*w > h$, too few individuals commit the act; if $x^* > 0$, there are additional enforcement expenses incurred. Hence, it must be that $s^* = h/p^*$, and the claim follows.

iii) If $h \geq p^*w$, then $s^*(h) = w$; also, for such h, $x^*(h)$ is at first zero: If $s^* < w$, then by part i, $x^* = 0$. But since $p^*s^* < p^*w \leq h$, it is socially beneficial to raise s, a contradiction. Thus $s^* = w$. Therefore, (A8) equals

$$\int_{p(x,y)w}^{\infty}(b-h)f(b)dbg(h)-x. \quad \text{(A9)}$$

The derivative of (A9) with respect to x is

$$-p_x w(pw-h)f(pw)g(h)-1. \quad \text{(A10)}$$

This is negative in a neighborhood of h above $h = p^*w$, so that $x^* = 0$ in the neighborhood. Q.E.D.

[17] Although I have assumed that $y^* > 0$, it is of interest to observe that this must be true if $E(h)$, the mean of h, is sufficiently large. Specifically, $y^* > 0$ if the derivative of (A6) evaluated at zero is positive. This derivative at zero is, from (A7), equal to $p'(0)wf(0)E(h) - 1$, which is positive if $E(h)$ is large enough.

Note

We assumed, recall, that $x^*(h)$ is positive for some h, and when this is so, $x^*(h)$ is determined by the condition that (A10) equals zero. The sign of the partial derivative of (A10) with respect to h may be positive or negative, so that $x^*(h)$ may rise or fall with h.

Proof of Proposition 4

The social problem for each h is to choose x and z to maximize

$$\int_{p(x)z}^{\infty} [b - h - \sigma p(x)z]f(b)dbg(h) - x. \tag{A11}$$

Let us now demonstrate two claims.

i) If $x^*(h) > 0$, then $z^*(h) = \bar{z}$: Assume otherwise, that $x^* > 0$ and $z^* < \bar{z}$. Raise z to $\bar{z}$ and lower x to x' such that $p(x')\bar{z} = p(x^*)z^*$. Then the integral in (A11) is unchanged—the same individuals commit the act *and* the expected social cost of punishment $\sigma p(x)z$ is unaltered—but enforcement effort is lower. Thus (A11) is higher, a contradiction.

ii) $x^*(h) = 0$ for all h sufficiently low: From part i, we know that if x^* is positive, x^* maximizes

$$\int_{p(x)\bar{z}}^{\infty} [b - h - \sigma p(x)\bar{z}]f(b)dbg(h) - x \tag{A12}$$

over x. Using (A12), we can show that $x^* = 0$ if h is sufficiently small.[18] Q.E.D.

Note

The derivative of (A12) with respect to x is

$$\{-p'(x)\bar{z}(p\bar{z} - h - \sigma p\bar{z})f(p\bar{z}) - \sigma p'(x)\bar{z}[1 - F(p\bar{z})]\}g(h) - 1. \tag{A13}$$

The first term of (A13) is the marginal gain due to increased deterrence, and the second term is the social cost due to imposing imprisonment with greater likelihood (F is the cumulative distribution function of f). If $x^*(h) > 0$, it is determined by the first-order condition that (A13) equals zero.[19]

If enforcement effort is general, the social problem is to choose y and $z(h)$ to maximize

$$\int_{0}^{\infty}\int_{p(y)z(h)}^{\infty} [b - h - \sigma p(y)z(h)]f(b)dbg(h)dh - y, \tag{A14}$$

and we assume as before that $y^* > 0$.

[18] An indirect argument demonstrates this. Consider the problem of maximizing social welfare (A12) assuming that $\sigma = 0$. It is clear that, for any $x > 0$, (A12) is higher if $\sigma = 0$ than if σ is positive. Hence, if $x = 0$ maximizes (A12) when $\sigma = 0$ for all h sufficiently low, $x = 0$ must maximize (A12) for such h when σ is positive as well. But when $\sigma = 0$, the problem of maximizing (A12) is identical in form to maximizing (A1) ($\bar{z}$ plays the role of w). For this problem we know that $x = 0$ is optimal for all h sufficiently small.

[19] I shall not note similar first-order conditions after subsequent propositions.

Proof of Proposition 5

i) $z^*(h) = 0$ for all h sufficiently small; $z^*(h) > 0$ for some h. If $0 < z^*(h) < \bar{z}$, then $z^*(h)$ increases with h: Given p, the social problem for any h is to maximize over z

$$\int_{pz}^{\infty} (b - h - \sigma pz) f(b) db. \tag{A15}$$

The derivative of this with respect to z is

$$-p(pz - h - \sigma pz) f(pz) - \sigma p[1 - F(pz)]. \tag{A16}$$

When $z = 0$, (A16) equals $phf(0) - \sigma p$. Hence, for h sufficiently small, (A16) is negative, and $z = 0$ is a local maximum; $z = 0$ can also be shown to be a global maximum.[20] Also, because, when $z = 0$, (A16) is positive for h sufficiently large, $z^*(h) > 0$ for such h. If z is an interior optimum, it is determined by the first-order condition

$$-p(pz - h - \sigma pz) f(pz) = \sigma p[1 - F(pz)]. \tag{A17}$$

Since the partial derivative of this with respect to h is $pf(pz) > 0$, $z^{*\prime}(h) > 0$.

ii) If $z^*(h) = \bar{z}$ for some h, it equals $\bar{z}$ for all higher h: If not, then (under the assumption that $z^*(h)$ is continuous), $z^*(h)$ must fall with h over some region, but this contradicts part i.

iii) $z^*(h) = \bar{z}$ for all h sufficiently high: If not, then part ii implies that $z^*(h) < \bar{z}$ for all h. Since, by part i, $z^*(h)$ is positive for all h sufficiently large, we have that $0 < z^*(h) < \bar{z}$ for all h sufficiently large. This leads to a contradiction. On one hand, (A17) must hold for h sufficiently large since $z^*(h)$ is an interior solution. On the other hand, $z^*(h)$ must approach a limit, say $\hat{z}$ as $h \to \infty$, since $z^*(h)$ is, by part i, increasing in h. But as $h \to \infty$, the left-hand side of (A17) tends toward $-p(p\hat{z} - h - \sigma p\hat{z}) f(p\hat{z})$, which grows unboundedly, whereas the right-hand side of (A17) tends toward $\sigma p[1 - F(p\hat{z})]$. Thus (A17) cannot hold as $h \to \infty$, a contradiction. Q.E.D.

[20] Since (A16) is negative when $h = 0$ and $z = 0$ and is continuous in h and z, (A16) must be negative for all h and z in some square $[0, \delta] \times [0, \delta]$, where $\delta > 0$. Hence, if there is a global maximum at a positive z at any h in $[0, \delta]$, z must be above δ. If $z > \delta$, then for any $h < p\delta$, individuals with b in $[h, p\delta]$ are discouraged from committing acts (the expected sanction is $pz > p\delta$) even though social welfare would be increased if they did commit harmful acts. Hence, there is a loss relative to first-best behavior of at least

$$\int_{h}^{p\delta} (b - h) f(b) db.$$

(There is also a loss due to the social cost of imposing sanctions.) But the only loss relative to first-best behavior if $z = 0$ is

$$\int_{0}^{h} (b - h) f(b) db.$$

For all h sufficiently small, this expression is dominated by the one above, so that $z > \delta$ cannot be optimal for such h. Hence, $z = 0$ must be the global optimum for all such small h, as claimed.

When enforcement effort is both general and specific, social welfare equals

$$\int_0^\infty \int_{p(x(h),y)z(h)}^\infty (b - h - \sigma pz)f(b)dbg(h)dh - \int_0^\infty x(h)dh - y, \tag{A18}$$

and we assume as before that $y^* > 0$ and that $x^*(h) > 0$ for some h.

Sketch of Proof of Proposition 6

I shall only sketch the arguments. Given what has been said, it will be apparent that the claims can be established, and it would be tedious to supply the details. Observe first that $x^*(h) > 0$ implies that $z^*(h) = \bar{z}$. The social problem for any h is to choose x and z to maximize

$$\int_{p(x,y)z}^\infty (b - h - \sigma pz)f(b)dbg(h) - x. \tag{A19}$$

Assume that $x^* > 0$ but that $z^* < \bar{z}$. Raise z to $\bar{z}$ and lower x so that pz is constant. Then the integral in (A19) is constant; since x is lower, (A19) is higher, a contradiction. Since $x^*(h)$ is zero until $z^*(h)$ is at its maximum, $z^*(h)$ is determined essentially as described in proposition 5 until $z^*(h)$ equals $\bar{z}$.

Proof of Proposition 7

Given h, the problem is to choose x, s, and z to maximize

$$\int_{p(x)(s+z)}^\infty [b - h - \sigma p(x)z]f(b)dbg(h) - x. \tag{A20}$$

I establish several claims about the solution.

i) If $x^*(h) > 0$, then $s^*(h) = w$: Assume otherwise, that $x^* > 0$ and $s^* < w$. Raise s to w and lower x to x' such that $p(x')(w + z) = p(x^*)(s^* + z)$. Then the integral in (A20) can only rise: the same individuals commit the act and the social cost of imposing imprisonment, $\sigma p(x')z$, falls if $z > 0$. Since enforcement effort is lower, (A20) is higher, a contradiction.

ii) If $x^*(h) > 0$, then $z^*(h) < \bar{z}$ is possible: Assume that $x^* > 0$ and that $z^* = \bar{z}$. Lower z slightly to z' and raise x to x' so as to keep the expected sanction constant. Thus the set of individuals who commit the harmful act is unchanged given z' and x'. However, expected imprisonment *falls:* Since, by part i, $s^* = w$, we have

$$p(x')(w + z') = p(x^*)(w + \bar{z}) \tag{A21}$$

or, equivalently,

$$p(x')z' = p(x^*)\bar{z} - [p(x') - p(x^*)]w; \tag{A21$'$}$$

the term $[p(x') - p(x^*)]w$ is positive since $x' > x^*$. Hence, $p(x')z' < p(x^*)\bar{z}$, as asserted. Therefore, the integral in (A20) rises by

$$\sigma[p(x') - p(x^*)]w[1 - F(p(x^*)(w + \bar{z}))]g(h). \tag{A22}$$

If σ is sufficiently high, (A22) will exceed the increase in enforcement effort, $x' - x^*$,[21] and (A20) will rise, a contradiction. Hence, $z^* < \bar{z}$ will hold. (It

[21] The derivative of (A22) with respect to x', evaluated at x^*, is $\sigma p'(x^*)w(1 - F)g(h)$; the derivative of the increase in enforcement effort is one. Hence, if σ is such that $\sigma p'(x^*)w(1 - F)g(h) > 1$, it is clear that the statement in the text is valid.

should be noted that the argument just supplied does not establish that $z^* < \bar{z}$ must hold, only that it can hold.)

iii) $x^*(h) = 0$ for all h sufficiently low: That $x^*(h) = 0$ if h is sufficiently small follows from an argument similar to that given above in the proofs of propositions 1 and 4. Q.E.D.

Proof of Proposition 8

i) $s^*(h) < w$ implies $z^*(h) = 0$; that is, imprisonment is not employed unless maximal monetary sanctions are: Given p, the problem for any h is to choose s and z to maximize

$$\int_{p(s+z)}^{\infty} (b - h - \sigma p z) f(b) db. \tag{A23}$$

Assume that $s^* < w$ but $z^* > 0$. Then increase s slightly and decrease z by the same amount, so that their sum is constant. This means that the same individuals commit the act; but since z is lower, the integrand is higher, so (A23) is higher, a contradiction.

ii) If $h < p^*w$, then $s^*(h) = h/p^*$ and $z^*(h) = 0$: From part i, we know that there exist two possibilities for the solution to (A23): that $s < w$ and $z = 0$, or that $s = w$ and $z \geq 0$. If $s < w$ and $z = 0$, the problem (A23) is the same as the problem with monetary sanctions, (A4). But for this problem, we know that the optimal s is h/p^* (which is less than w). On the other hand, if $s = w$, then (A23) is clearly less than if $s = h/p^*$ and $z = 0$. (If $s = w$, fewer individuals for whom $b > h$ commit the act since $p^*w > h$; if they do commit the act, the integrand will be lower if $z > 0$.) Hence, $s^* = h/p^*$ and $z^* = 0$, as claimed.

iii) If $h \geq p^*w$, then $s^*(h) = w$: If $s^* < w$, then, by part i, $z^* = 0$. Hence, (A23) becomes (A4). But since $p^*s^* < p^*w \leq h$, increasing s increases (A4). This contradicts the supposition that s^* was optimal.

iv) $z^*(h) = 0$ in an interval $[p^*w, h']$, where $h' > p^*w$: From part iii, we know that, for any $h \geq p^*w$, (A23) is

$$\int_{p^*(w+z)}^{\infty} (b - h - \sigma p^* z) f(b) db. \tag{A24}$$

The derivative of (A24) with respect to z is

$$-p^*[p^*(w + z) - h - \sigma p^* z] f(p^*(w + z)) - \sigma p^*[1 - F(p^*(w + z))]. \tag{A25}$$

Evaluating (A25) at $h = p^*w$, we obtain $-p^*(p^*z - \sigma p^*z) f(p^*(w + z)) - \sigma p^*[1 - F(p^*(w + z))]$. At $z = 0$, this equals $-\sigma p^*[1 - F(p^*w)]$, so that $z = 0$ is a local maximum. By continuity, (A25) is negative at $z = 0$ for h in a neighborhood above $h = p^*w$, so that $z = 0$ is a local maximum in such a neighborhood. By an argument analogous to that in note 20, $z = 0$ can also be shown to be a global maximum in a neighborhood above $h = p^*w$.

On the other hand, (A25) evaluated at $z = 0$ is $-p^*(p^*w - h) f(p^*w) - \sigma p^*[1 - F(p^*w)]$. Since this is positive for h sufficiently large, $z^*(h) > 0$ for such h. Q.E.D.

Sketch of Proof of Proposition 9

First, $s^*(h) < w$ implies $z^*(h) = 0$ and $x^*(h) = 0$; that is, neither imprisonment nor specific enforcement effort is employed unless maximal monetary sanctions are imposed. This is true because the social problem for any h is to choose x, s, and z to maximize

$$\int_{p(x,y)(s+z)}^{\infty} (b - h - \sigma pz) f(b) db g(h) - x. \tag{A26}$$

If $s^* < w$ but $z^* > 0$, then by increasing s slightly and reducing z, so that $s + z$ is constant, (A26) can be raised. Hence $z^* = 0$. Second, $s^*(h) < w$ implies $x^*(h) = 0$, for if $s^* < w$ and $x^* > 0$, then by increasing s slightly and reducing x, ps can be held constant. Thus the integral in (A26) will be unchanged (for z^* must equal zero, as just shown), and (A26) will therefore rise since x is lower. Hence, $x^* = 0$.

Also, with the arguments in the proof of proposition 8, it may be shown that $s^*(h)$ equals $h/p(0, y^*)$—so that the expected sanction is h—over the interval $[0, p(0, y^*)w]$. Beyond this interval, monetary sanctions are maximal, and imprisonment and specific enforcement effort become positive.

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